Damage localization as a possible mechanism underlying earthquakes

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Introduction

Failure of solids is a highly rate-dependent and non-linear process. Sometimes failure is noted as a process of reduction of dimension, like a roughly two-dimensional fracture surface formed in a three-dimensional body of solids. This is also known as localized failure, though distributed microdamages developed. Usually, localized failure occurs suddenly, hence becomes more dangerous, like earthquake. According to statistical evolution of microdamage, this paper intends to reveal what intrinsic factor in solids governs the localized failure.

Basic model and equation

We have established a fundamental equation of microdamage in the light of statistical mesoscopic damage mechanics (Bai et al., 1991, 1997). This is the evolution of microdamage in phase space p_i

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{I} \frac{\partial (n \cdot P_i)}{\partial P_i} = n_N - n_A \quad , \tag{1}$$

where n is the number density of microdamage in phase space and t is time. $P_i = \dot{p}_i$ are the rates of variables p_i. n_N and n_A are nucleation and annihilation rate densities of microdamage respectively.

We investigate such an element of solid that is large enough to contain a number of microdamages with different sizes but is small enough to be handled as a point macroscopically. In this way, we actually examine a phase space c, x, i.e., $p_1 = c$ and $p_2 = x$, where c is current size of microdamage and x is macroscopic spatial coordinates. This is a revised version of continuum to include meso-structual variables,

$$\frac{\partial n}{\partial t} + \frac{\partial nA}{\partial c} + \frac{\partial nv}{\partial x} = n_N - n_A \quad , \tag{2}$$

where $A = \dot{c}$ and v is particle velocity. Suppose that τ denote the average failure volume of a microdamage with current size c. Continuum damage should be defined by

$$D = \int_0^{\inf} n(t, c, x) \tau dc \quad . \tag{3}$$

It describes the fraction of damage in unit current volume. After multiplying equation 2 by τ and integrating it, we obtain the evolution equation of damage field

$$\frac{\partial D}{\partial t} + \nabla \cdot (Dv) = f \quad , \tag{4}$$

$$f = \int_0^{\inf} n_N \tau dc + \int_0^{\inf} (nA\tau' - n_a \tau) dc \quad , \tag{5}$$

where $\tau' = \frac{\partial \tau}{\partial c}$ is defined as the dynamic function of damage (DFD), which is uniquely dependent on mesoscopic dynamics of microdamage. In this formulation, continuum damage is described by the field variable $D = D(t, x)$. Since the mesoscopic dynamics of microdamage are governed by continuum variables, such as stress and continuum damage, DFD f can be assumed to be $f(\sigma, D)$ as an approximation.

Criterion for damage localization

The condition for damage localization is supposed to be

$$\left(\frac{\partial \left(\frac{\partial D}{\partial y} \right)}{\partial t} \right) / \left(\frac{\partial D}{\partial y} \right) \geq \left(\frac{\partial D}{\partial t} \right) / D \quad . \tag{6}$$

Under the approximation of one dimensional quasistatic and small deformation, a lower bound for damage localization can be deduced and expressed by

$$f_d \geq f/D \quad , \tag{7}$$

where $f_d = \frac{\partial f}{\partial D}$. Clearly, the compound damage f_d is the motivation of damage localization. Since the dynamic function of damage is usually concave, especially at the late stage of damage evolution owing to coalescence of microdamages, there is almost always a tendency to damage localization, see Fig. 1.

Inversion to dynamic function of damage

Dynamic function of damage (DFD) is the core in damage evolution, but this function deeply roots in the mesoscopic mechanisms of damage evolution. From engineering point of view, we need some operation approach to it. In the light of the damage field equation, we developed an inverse method to cope with the problem. When the evolutions of stress, strain and damage, $\sigma(T), \Theta(T)$ and $D(T)$ in Lagrangian coordinates (T, X) have been measured in practice or calculated in simulation once can derive the evolutions of the dynamic function of damage $f(T)$ as follows,

$$f = \frac{\partial D}{\partial T} + \frac{D\dot{\Theta}}{1+\Theta} \quad . \tag{8}$$

Now, we choose a set of samples with different initial damage D_0 from the same material. Then all the evolutions become binary functions, like $\sigma(T, D_0, D(T, D_0)$ and $f((T, D_0)$. After transformation of the independent variables T and D_0 into σ and D, the dynamic function of damage f can be expressed as binary function $f(\sigma, D)$. This is the required dynamic function of damage.

Example: a numerical prediction

We carried out a set of numerical simulations to examine the damage accumulation and to predict the occurrence of failure in terms of the criterion for damage localization.

Fig. 2(a) shows the microdamage pattern at the moment when the criterion for damage localization 7 gives an alarm. Fig. 2(b) demonstrates the corresponding load - displacement (the upper one) and damage - displacement (the lower one) curves until this moment. The cross (+) in the Figure indicates the alarm for the coming damage localization. But, there are nearly no hints of damage localization at all in the microdamage pattern (Fig. 2). When one compares Figure 2 with successive damage patterns, it becomes clear that localized damage does develop after the alarm. Figure 3 shows the final failure state of the process. Also, it shows clearly that the prediction for damage localization does provide a proper warening. Rtelevant animation of the evolution of the damage field demonstrates the process much more clearly. So, we thought that the procedure and the criterion for damage localization may be used to earthquake understanding.

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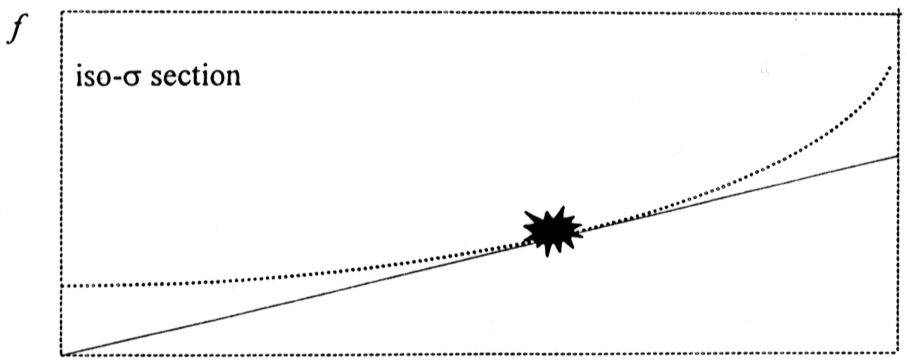

Figure 1: Schematic lower bound for damage localization $f_d \geq f/D$

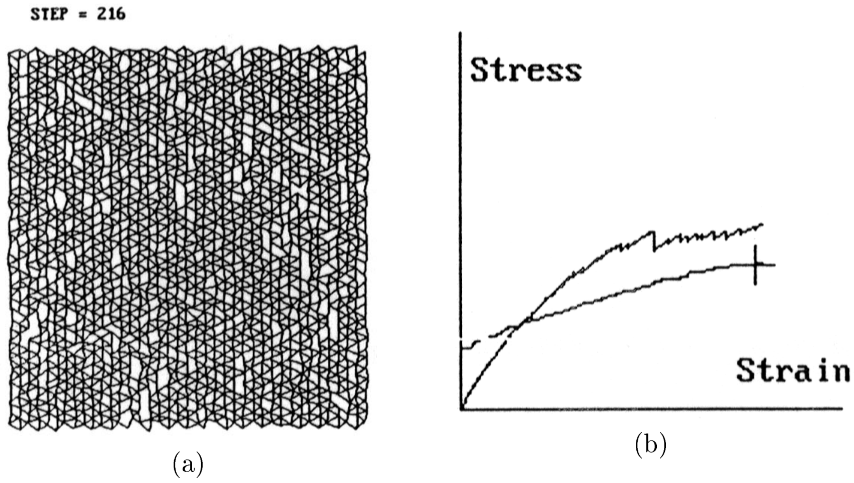

Figure 2: The microdamage pattern at the moment when the criterion for damage localization (6) gives alarm (a) and the corresponding load - displacement (the upper one) and damage - displacemant (the lower one) curves (b). The cross (+) indicates the alarm for the coming damage localization.

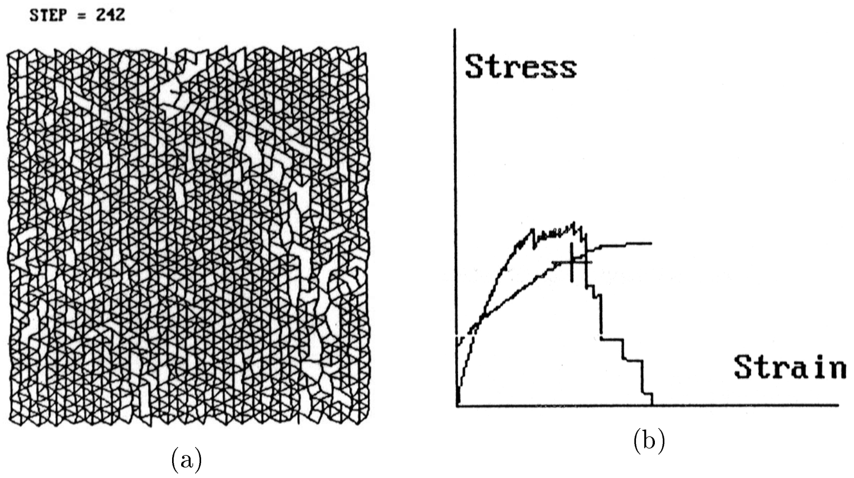

Figure 3: The final failure state (a) and the corresponding load - displacement (the upper one) and damage - displacemant (the lower one) curves (b).The cross (+) indicates the alarm for the coming damage localization.