Hybrid modelling of coupled pore fluid-solid deformation problems

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Abstract

A hybrid formulation for coupled pore fluid-solid deformation problems is proposed. The scheme is hybrid in the sense that we use a vertex centered finite volume formulation for the analysis of the pore fluid and a particle method for the solid. The pore fluid occupies the same space as the solid particles. The solid particles are viewed as carriers of mass and momentum. The finite volume mesh is generated by Delaunay triangulation. Each triangle possesses an initial porosity. Changes of the porosity are specified by the motions of the mass centres of the solid particles. Each particles includes in its momentum balance the resultant of the pore fluid surface traction. The volume changes of the solid material are considered in the mass balance of the pore fluid. The deformation of the solid influences the permeability of the matrix material.

Introduction

We have elaborated on the application of a particle based modelling scheme to solid deformation and fracture problems. Many important geophysical processes as well as a large number of geotechnical problems embody coupling between solid and fluid momentum transfer; sometimes heat and reactive mass transfer have to be considered as well. Those interactions, this is a key issue as geo-dynamics, must be included into the fault zone processes modelling based on the particle method. In this report we present a hybrid formulation for coupled pore fluid-solid deformation problems. The scheme is hybrid in the sense that we use a vertex centered finite volume formulation for the discretisation of the pore fluid and a particle method for the solid. The coupling between pore fluid flow and solid deformation occurs in three ways: 1) The volume changes of the solid are considered in the mass balance of the pore fluid; 2) Each particle includes in its momentum balance the resultant of the pore fluid surface traction; 3) The deformation of the solid influences the porosity which directly relates to permeability of the matrix material. Fractures modelled by bond breakages in particle method are reflected as zones of high permeability. The potential of this new model is illustrated by means of example solutions related to the fault zone processes in rocks.

Implementation

In the absence of a pore fluid, the momentum balance (Sakaguchi and Mühlhaus, 1997 [1]) reads:

$$\sum_{contacts} \boldsymbol{F}^{ji} + \boldsymbol{\gamma}^i - m^i \dot{\boldsymbol{v}}^i = 0 \quad , \tag{1}$$

where $\boldsymbol{F}^{ji}$ is the contact force between the particles i and j, $\boldsymbol{\gamma}^i$ is the weight vector of the particle i, m^i is the particle mass, $\dot{\boldsymbol{v}}^i$ is the translational velocity of the mass centre and the overdot designates time derivatives. To consider the influence of pore fluid flow we replace the weight vector $\boldsymbol{\gamma}^i$ by

$$\boldsymbol{\gamma}^i \Longrightarrow \boldsymbol{\gamma}^i + \sum_{surface} \boldsymbol{P}^{ji} \quad , \tag{2}$$

$$\boldsymbol{P}^{ji} = \frac{1}{2} s^{ji}(1 - \phi^{ji})(P^j + P^i)\boldsymbol{n}^{ji} \quad , \tag{3}$$

where P^i , P^j are pressures around the particles i and j, s^{ji} is the length of the connecting line between the mass centres of the triangular elements adjacent to the side ji, ϕ^{ji} is the average porosity of these elements and $\boldsymbol{n}^{ji}$ is the unit vector into the direction of the connecting line between the mass centres of the particles i and j. Obviously, if P is uniform then $\sum_{surface} \boldsymbol{P}^{ji} = 0$; if $gradP = -\rho g$ where ρ is the density of the pore fluid and g is the gravitational constant then $\sum_{surface} \boldsymbol{P}^{ji}$ is equal to the buoyancy force exerted by the pore fluid upon particle i. Equations (2) and (3) hold also in case of gas-solid interactions, in connection with the simulation of blasting operations for instance. Differences exist however in the form of the constitutive relationships. We assume Darcy's law for the average pore fluid flux (Detournay and Cheng, 1993 [2]). For simplicity we have assumed that the compressibility of the solid is negligible as compared to that of the drained bulk material, *i.e.* Biot's coefficient $\alpha = 1$. Then, for an incompressible pore fluid the rate of the pore pressure is controlled by the difference between the divergence of the pore fluid flux vector and the volume change of the solid. In the current version of the code we assume that the normal components of the fluid fluxes on the sides ij, jk and ki are equal to a scalar permeability times the normal component of the hydraulic head.

The three permeabilities are different in general and depend on the current extension of the sides ij, jk and ki. If the extension of the side ij, say, reaches a critical value then bond breakage between the particles i and j takes place and the permeability of that side K^{ij} is drastically increased to simulate the existence of cracks. However, such increases of the permeability alter severely the increment of the time step in calculation so that adaptable time stepping is required. It is well know that explicit integration schemes are usually not suitable for diffusion type problems such as Darcy flow. The pore fluid part of the numerical problem is therefore newly formulated in explicit-implicit hybrid form so that the explicit character of the particle method is retained.

Acknowledgments

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References

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