# Evolution induced catastrophe of material failure

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## Introduction

Earthquake is a kind of failure. Though material failure is a complex phenomenon, it is usually supposed that its essential features may be universal. But, neither percolation nor renormalization group theories, successfully used in equilibrium transition, describes failure properly, owing to its nonlinear and nonequilibrium nature. Here, by making use of a chain with non-linear evolution, we intend to present some universal features of failure. We call this model as EIC (Evolution Induced Catastrophe). We found that some minor mesoscopic differentiation can eventually induce macroscopic failure of materials, owing to nonlinear evolution far from equilibrium.

## Evolution induced catastrophe (EIC)

Fig. 1 shows an example demonstrating this evolution induced catastrophe phenomenon. Suppose that under a constant load random nucleation of microcracks occurs sequentially in a sample. The coalescence of the microcracks is governed by a deterministic non-linear law. Before the last nucleation of microcrack (No. 853), there are only distributed microcracks and separated linking microcracks. The sample as a whole remains globally stable (GS). However, the last nucleation of microcrack, though random, triggers a cascade of coalescence of microcracks and leads to eventual failure. We call this phenomena as EIC (evolution induced catastrophe). Actually EIC is defined as an abrupt transition from the gradual accumulation of damage to a catastrophe explosively. As shown above, two kinds of evolution modes appear according to their final states. These are globally stable mode (GS mode) and evolution induced catastrophic mode (EIC mode). Then, one must ask how to distinguish the two modes beforehand. This is to say, what is the boundary between the two modes in phase space?

## Trans-scale sensitivity

The mechanism underlying the two modes seems to be the sensitivity of macroscopic failure to some details of the meso-scopic pattern. We adopted an ensemble of one-dimensional chains to reveal its statistical feature. It is found that there is always a sensitive zone for non-linear evolution law, except for an overall averaging model. This sensitive zone is shown as a transition region in the failure probability $\Phi_N(p_o, \sigma_0)$, see Fig. 2.. In the sensitive zone, a slight meso-scopic change in the chain may lead to a significant macro-scopic consequence, namely the transition for GS to EIC of the chain. So, this is a trans-scale sensitivity. As a proverb said that the last straw breaks the camel's back. The EIC model describes this clearly.

From the viewpoint of failure prediction, we are mostly interested in the trans-scale sensitivity leading the transition from GS to EIC. But, how to depict the trans-scale sensitivity quantitatively? There are two ways to look at the sensitivity at least. These are to examine if a single jump and pair exchange of the states of a chain can lead to the transition from GS to EIC.

## Sensitivity due to single jump

We define a transition probability $\Psi_N(p_o, \sigma_0)$ to express the probability of jumps from GS to EIC, merely owing to a stochastic increment of one broken site in a chain. It was found that there is a relationship between the transition probability $\Psi_N(p_o, \sigma_0)$ and the failure probability $\Phi_N(p_o, \sigma_0)$

$$\Psi_N(p_o, \sigma_0) = -\frac{1}{N}\frac{\partial \ln(1 - \Phi_N(p_o, \sigma_0))}{\partial p_0} \quad .$$

It is found that the transition region of $0 < \Psi < 1$ is exactly in the transitional region of the failure probability $\Phi_N(p_o, \sigma_0)$, see Fig. 3.

## Sensitivity due to pair differentiation

The second possible sensitive configurations may come from the slightest pair exchange. That is to say, there is only one pair of neighboring sites in a chain with different options. So, in this situation, the fraction of initial damage remains the same. The failure probability with the pair exchange has double peaks, see Fig. 4. One peak corresponds to no qualitative change in final states, namely GS remains GS and EIC remains EIC. But, the other peak indicates the non-zero probability of the transition from GS to EIC due to the pair exchange. Though the probability is small, but not negligible.

## Similarly between EIC and earthquake

From the study on the EIC model, we feel that there seem to be some similarities between the EIC model and earthquake. Firstly, earthquake may essentially result from the evolution of faults an appear to be triggered by some minor event eventually. Secondly, earthquake seems to be sensitive to the faults configuration, too. This might be the difficulty of the prediction of earthquake. If so, perhaps, we should pay more attention to the statistical prediction according to the evolution of damage pattern, rather than individual cracking.

This work was supported by National Natural Science Foundation of China (No. 19732060) and the National Fundamental Research Project "Nonlinear Science".

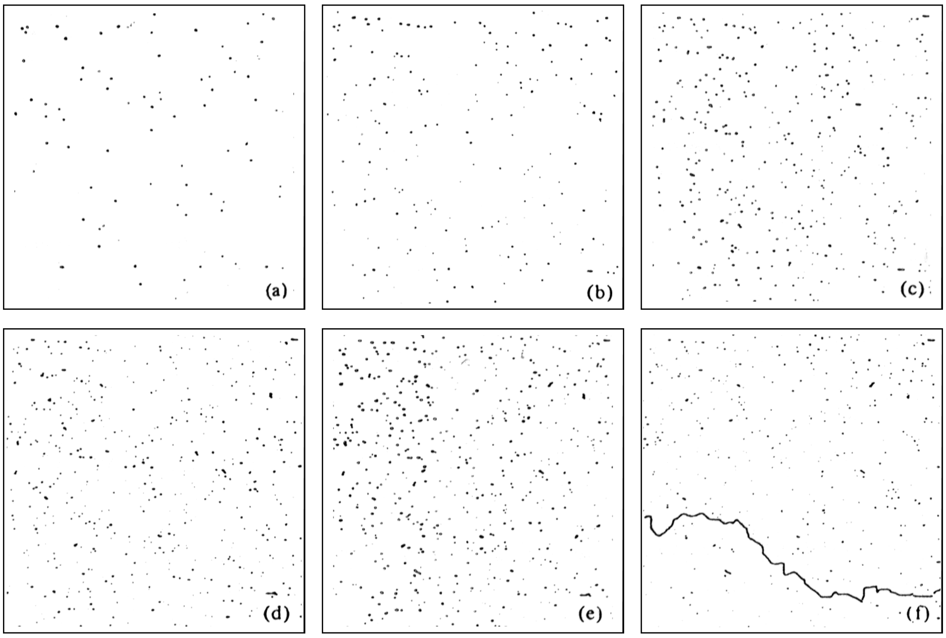



Figure 1: Computer simulation of EIC. The numbers of nucleated microcracks are (a) 100, (b) 300, (c) 500, (d) 700, (e) 831, (f) 832.

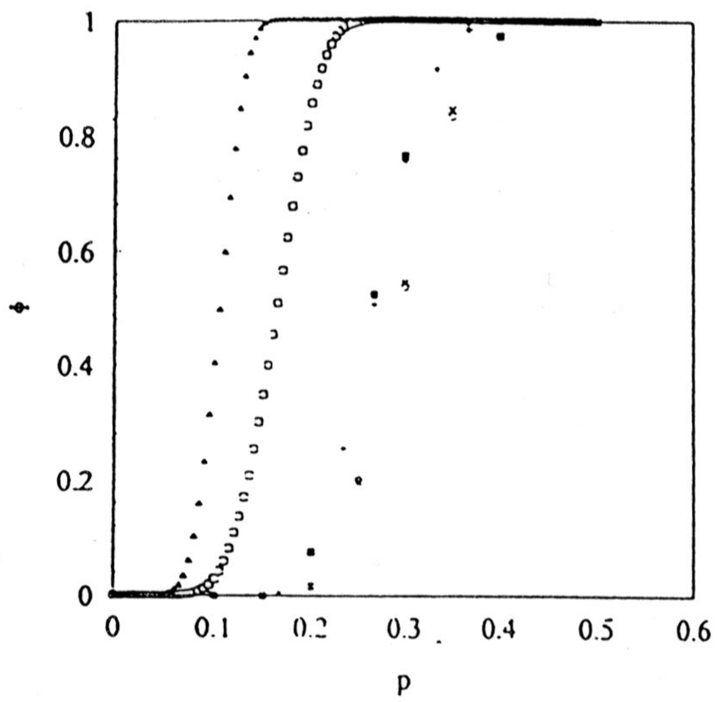
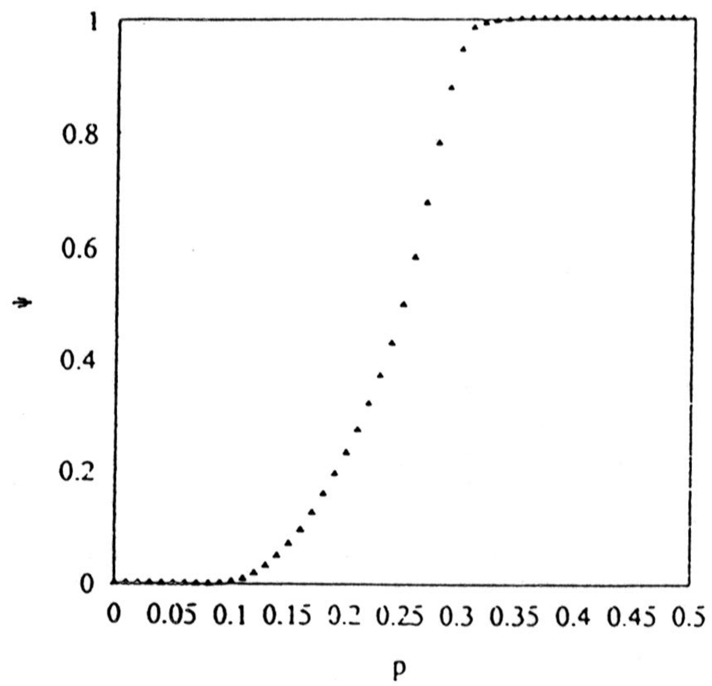
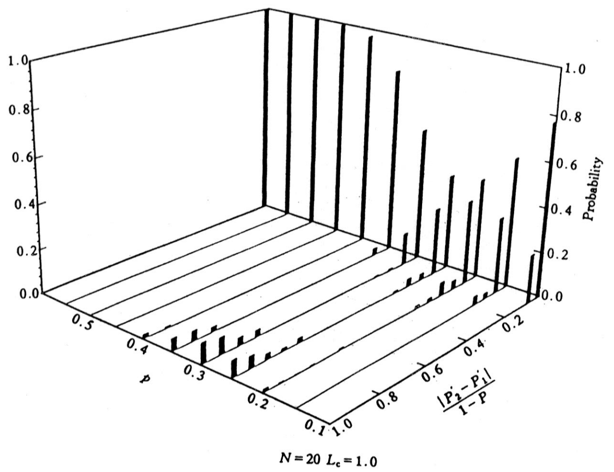



Figure 2: Failure probability $\Phi$. $L_c = 1.0$ Slice method (○) N=20; (+) N=30; (□) N=200; (▲) N=2000. Exact method: (x) N=20; (■) N=30. (-) Mean field theory




Transitional probability $\Psi$. N=100. $L_c$=1.0




The probabilty of difference of final fraction for samples with neighbouring initial stages, $L_c = 1.0$.

## References

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