Evolution of contacting rock surfaces and a slip- and time-dependent fault constitutive law

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Abstract

By integrating effects of microscopic interactions between statistically self-similar fault surfaces, we succeeded in deriving a slip- and time-dependent fault constitutive law that rationally unifies the slip-dependent law and the rate- and state-dependent law (Aochi and Matsu'ura, 1998[1]). In this constitutive law the slip-weakening results from the abrasion of surface asperities that proceeds irreversibly with fault slip. On the other hand, the restoration of shear strength after the arrest of fault slip results from the adhesion of surface asperities that proceeds with contact time. The slip- and time-dependent law can explain three basic experimental results; slip-weakening in high-speed slip, $\log t$-healing in stationary contact, and velocity-weakening in steady state slip. Another interesting property expected from the unified constitutive law is the gradual increase of the critical weakening displacement D_c with stationary contact time.

Introduction

The fault constitutive law is one of the basic equations governing the process of earthquake generation. At present we have two different types of laboratory-based constitutive law, one of which is the slip-dependent law (Ohnaka et al., 1987[6]; Matsu'ura et al., 1992[4]), and another is the rate- and state-dependent law (Dieterich, 1979[3]; Ruina, 1983[9]). So far these two constitutive laws have been regarded as incompatible concepts. We think, however, they are complements each of the other, because the former is based on rather high-speed experiments, while the later is based on very low slip-rate experiments. In this paper, first, a slip- and time-dependent fault constitutive law is theoretically derived by integrating effects of microscopic interactions (abrasion and adhesion) between statistically self-similar fault surfaces. Second, it is demonstrated that the new constitutive law unifies the slip-dependent law and the rate- and state-dependent law in a rational way.

Formulation of a slip- and time-dependent constitutive law

We consider a two-dimensional anti-plane fault on $x-z$ plane. Fault surfaces are irregular and have statistical self-similarity (Brown and Scholz, 1985[2]; Power et al., 1987[7]), and so we represent them by the superposition of surface asperities with various wave lengths. We use wavenumber k instead of wavelength $\lambda = 2\pi/k$, and introduce the complex Fourier components $Y(k)$ of surface topography. Then the statistical self-similarity of fault surfaces can be expressed by the following power spectral density distribution.

$$|Y(k)|^2 = \begin{cases} Ak_c^{-3} & 0 \le k < k_c \\ Ak^{-3} & k_c \le k < k_0 \\ 0 & k_0 \le k \end{cases} \quad . \tag{1}$$

Note that the upper fractal limit $\lambda_c = 2\pi/k_c$ is a key parameter controlling the slip-weakening behavior.

With the progress of fault slip, the state of contact changes as illustrated in Fig.1(a). The fault slip is always accompanied by abrasion of surface asperities, and then it leads to a decrease in asperity's amplitudes $|Y(k)|$. Matsu'ura et al. (1992) have described this process as

$$\frac{\partial}{\partial w}|Y(k;w)| = -\alpha k |Y(k;w)| \qquad \alpha > 0 \quad , \tag{2}$$

where w is the amount of slip and α is an abrasion rate. The above equation means that the rate of abrasion with slip is in proportion to the wavenumber k and the current asperity's amplitude $|Y(k)|$.

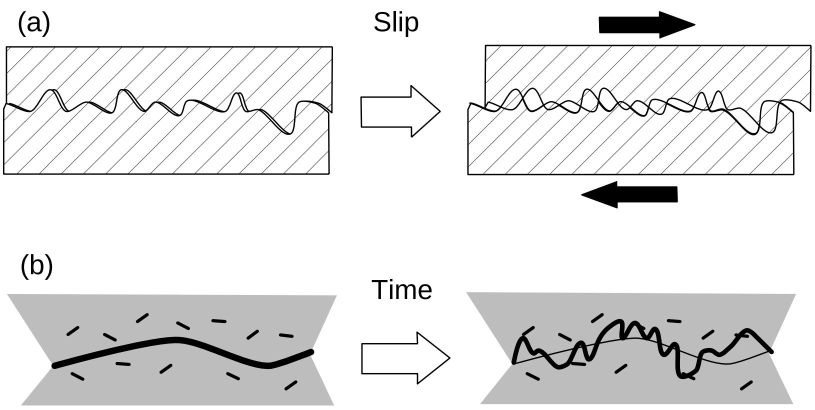


Figure 1: Schematic diagram showing the change in contact state between irregular rock surfaces with the progress of fault slip (a) and the process of adhesion and subsequent wear (b).

After the arrest of slip, the fault surfaces are in stationary contact after the arrest of slip. During the stationary contact, adhesion of fault surfaces proceeds gradually with time, and sticked portions become stronger than the preexisting weak interfaces around the fault surface, as shown in Fig.1(b). Then forthcoming shear rupture would occur along one of the preexisting weak interfaces (Rabinowicz, 1995[8]). Therefore, the adhesion and subsequent wear may be regarded as the process of reproducing fractal fault surfaces. We describe this process by the differential equation

$$\frac{\partial}{\partial t}|Y(k;t)| = \beta k^2(|\bar{Y}(k)| - |Y(k;t)|) \qquad \beta > 0 \quad , \tag{3}$$

where t is time, β is an adhesion rate with the same dimension as the diffusion coefficient, and $|\bar{Y}(k)|$ is a recoverable maximum amplitude.

Taking both effects of abrasion and adhesion into consideration, we can rationally extend the slip-dependent constitutive law derived by Matsu'ura et al. (1992) as follows:

$$\tau(w,t) = c\left[\int_0^\infty k^2 \sin^2(kw/2)|Y(k;w,t)|^2 dk\right]^{1/2} + \tau_c \quad , \tag{4}$$

$$d|Y(k;w,t)| = -\alpha k|Y(k;w,t)|dw + \beta k^2(|\bar{Y}(k)| - |Y(k;w,t)|)dt \quad . \tag{5}$$

The first equation, derivation of which is given in Matsu'ura et al. (1992), states that the shear strength τ of fault is determined by the power spectral density distribution of surface topography, changing with fault slip w and contact time t. The constant term τ_c indicates a frictional strength of fault when rocky surfaces are in the state of indirect contact. The evolution of surface topography with w and t is governed by the second equation, as we have explained already.

Properties of the slip- and time-dependent constitutive law

In order to examine the properties of the slip- and time-dependent constitutive law defined by equations (4) and (5), we consider three extreme cases. The first is the case of high-speed slip ($V \equiv dw/dt >> 0$). In this case we can neglect the effect of adhesion, and so the fault surfaces evolve as a function of slip w. Fig. shows the change of shear strength $(\tau - \tau_c)$ normalized by

$$\Delta\tau_0 \equiv c\left[\int_0^\infty \frac{1}{2}k^2|Y_0(k)|^2 dk\right]^{1/2} \quad , \tag{6}$$

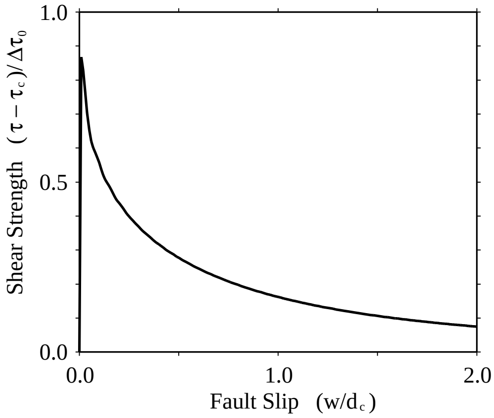
which can explain the slip-weakening behavior observed in laboratory experiments, as Matsu'ura et al. (1992) [4] have already demonstrated.

The second is the case of stationary contact ($V = 0$). In this case we can neglect the effect of abrasion. The fault surfaces evolve to $|\bar{Y}(k)|$ with time t. Fig. shows the $\log t$ recovery of shear strength, which is consistent with experimental results.

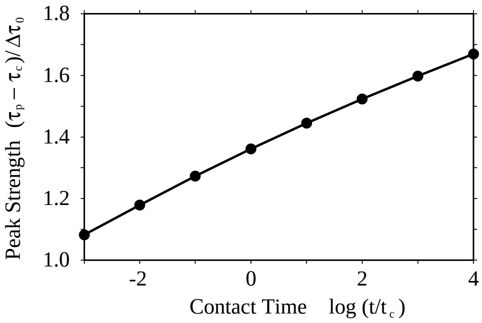
The third is the case of steady state ($d|Y(k)| = 0$) at a constant slip velocity V. We show the relation of shear strength τ_{ss} and slip velocity V in Fig., which can explain the velocity-weakening behavior observed in laboratory experiments.

Discussion

We theoretically derived a slip- and time-dependent constitutive law by considering the evolution of contacting rock surfaces due to abrasion and adhesion. This constitutive law can consistently explain the three basic experiments; slip-weakening in



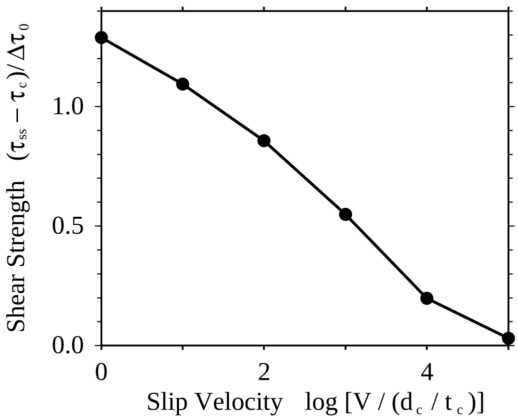
Change in shear strength with fault slip in high-speed slip. The initial values of k_c (upper fractal limit) and k_0 (lower fractal limit) are taken to be 10 and 10^4, respectively. The strength τ is normalized by the initial strength rise $\Delta\tau_0$ defined by equation (6), and the fault slip w by the characteristic weakening displacement $d_c \equiv (\alpha k_c)^{-1}$.



Change in peak strength with time in stationary contact. The contact time t is normalized by the characteristic healing time $t_c = (\beta k_c^2)^{-1}$. The other parameters are the same as in Fig..

high-speed slip, $\log t$-healing in stationary contact, and velocity-weakening in steady state slip.

Another interesting fault property expected from this law is the increase of the critical weakening displacement D_c with contact time t. We consider the change of constitutive relation during a simple slip-stick-slip process. For simplicity, we put $dt = 0$ during slip and $dw = 0$ during stick. Fig.2 shows the evolution of constitutive relation after the arrest of the first slip. We can see that the peak strength increases with the time t. Furthermore we can see that D_c also increases with the stick time. This phenomena may be attributed to the gradual recovery of large-scale fractal structure of fault surfaces with contact time, which has been confirmed by Nakatani (1997)[5] through slip-hold-slip tests in laboratory.



Theoretical slip-velocity dependence of shear strength in steady-state slip. The slip velocity V is normalized by d_c/t_c.

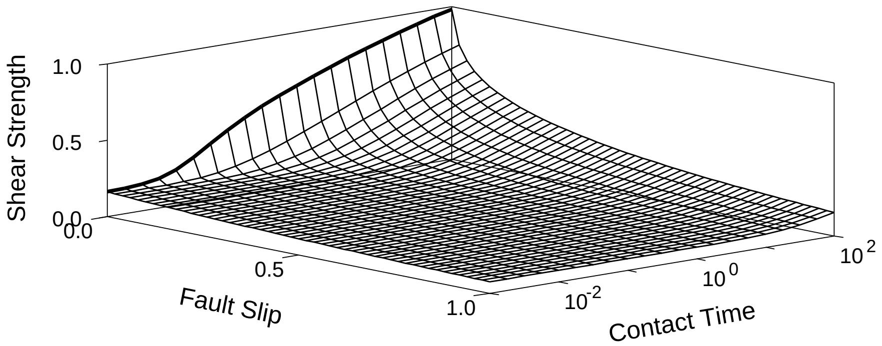


Figure 2: Evolution of constitutive relation after the arrest of a high-speed fault slip. The shear strength τ, the fault slip w, and the contact time t are normalized by τ_0, d_c, and t_c, respectively.

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