Sample-specificity and predictability of material failure

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Abstract

It is found that brittle failure of materials with distributed microcracks or with complicated microstructures shows sample-specificity. This may change the formulation of failure prediction, e.g. to adopt probabilistic prediction. For each failure event, there may be some statistical precursors to the eventual failure, which can be used to make prediction. This might be one mechanism governing earthquake, which can be understood as a local failure of geological media, in a sense.

Both earthquake prediction and failure prediction of disordered brittle materials are difficult and complicated problems. The complexity comes partly from the followings. Firstly, they are usually the phenomena far from equilibrium. Secondly, the collective effects of disordered inhomogeneities on multi-scales play important roles. Here, we discuss a distinct feature of failure: sample-specificity, i.e. a great diversity of macroscopic failure threshold under the same macroscopic conditions. If so, a deterministic macroscopic prediction becomes impossible. Then we turn to statistical description, e.g. failure probability. The underlying mechanism of sample-specificity is that the effects of disorder on smaller scales can be strongly enhanced during nonlinear evolution and become significant on large scale. This is a sensitivity linking different scales. A possible approach to this problem is to analyze the evolution of the statistical ensemble for the nonlinear dynamical system.

As an example, we consider the damage and failure of a bundle of fibers stringing a one-dimensional chain. A fiber is regarded as a mesoscopic unit, and the chain is regarded as a macroscopic system. Mesoscopically, the system is described by damage pattern X and governing field pattern, e.g. the stress pattern, $\sum$. $\sum$ is determined by X, and the evolution of X is governed by $\sum$. Then, the mesoscopic dynamics is given by a coupling-pattern evolution model. Macroscopically, the system is described by the damage fraction p (its initial value is denoted by p_0) derived from pattern X and the nominal stress σ_0 derived from pattern $\sum$. The mesoscopic disorder may come from the random distribution of initial broken units or the values of unit strength. In order to analyze the effects of mesoscopic disorder, we construct a phase space based on damage patterns, and examine the evolution of statistical ensemble by using a method called slice-sampling. This method is to take a series of two-dimensional

slice through phase space stochastically, and let the phase points on each slice to be interelated.

The typical result of failure probability $\Phi_N(p_0, \sigma_0)$ is shown in Fig. 1, where N is the number of units in a chain representing the scale span between macroscopic system and mesoscopic unit. A distinct feature is that there is a transitional region with $0 < \Phi < 1$ in the macroscopic parameter space p_0, σ_0. In the transitional region, globally stable modes and catastrophic failure modes coexist. So, the appearance of transitional region indicates sample-specificity, i.e. macroscopic failure cannot be determined by macroscopic parameters uniquely but depends on the details of mesoscopic pattern.

In practical systems, the typical scale span between macroscopic and mesoscopic scales is in the range of $N \sim 10^2 - 10^6$. It is interested to examine the scale effect of the transitional region. The results are shown in Fig. 2, where p_t and Δ are the characteristic position and width, respectively, of transitional region in p_0-direction and $\bar{\sigma_f}$ and $\Delta\sigma_f$ are those in σ_0-direction. Approximately, the scale effect demonstrates scaling laws, which results in slow decreasing of Δ/p_t and $\Delta\sigma_f/\bar{\sigma_f}$ with N. For cluster load-sharing model, from $N = 10^2$ to 10^6, $\Delta\sigma_f$ decreases from 0.105 to 0.069 and $\Delta/p_t \cong 0.19$ remains nearly a constant. These results imply that sample-specificity does play an important role in practical failure.

In practice, statistical prediction of materials failure may be insufficient. A precursor hunting shows that a clue for failure prediction may be hidden in the fluctuations of governing field. In fact, a menu field model leads to a deterministic macroscopic prediction. For a model with stress fluctuations, macroscopic failure exhibits sample-specific behaviour and the averaging failure threshold is much lower than that for mean field model. Fig. 3 shows the ensemble probability distribution $\pi(\Theta)$ of maximum standard deviation Θ in stress pattern during evolution. This is a well separated double peaks distribution, the lower and higher peaks come from globally stable modes and catastrophic failure modes, respectively. In other words, the maximum level of stress fluctuations for failure modes is much higher than that for globally stable modes. So, maybe we can set up a warning level for stress fluctuations to predict the appearance of catastrophic failure. Of course, this is still an open question.

In summary, based on ensemble statistics for simple dynamical system, we investigated some universal behaviors related to multi-scales effects far from equilibrium. These behaviors may be important in practical systems.

This work was supported by National Natural Science Foundation of China (No. 19732060) and the National Fundamental Research Project "Nonlinear Science".

References

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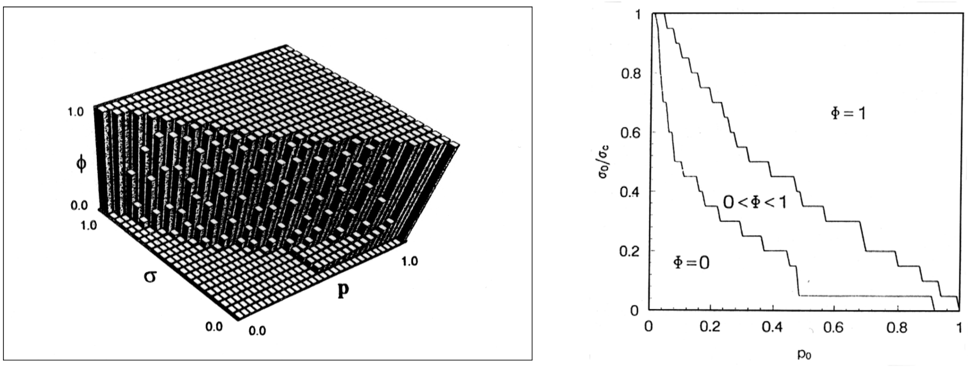

Figure 1: Failure probability.

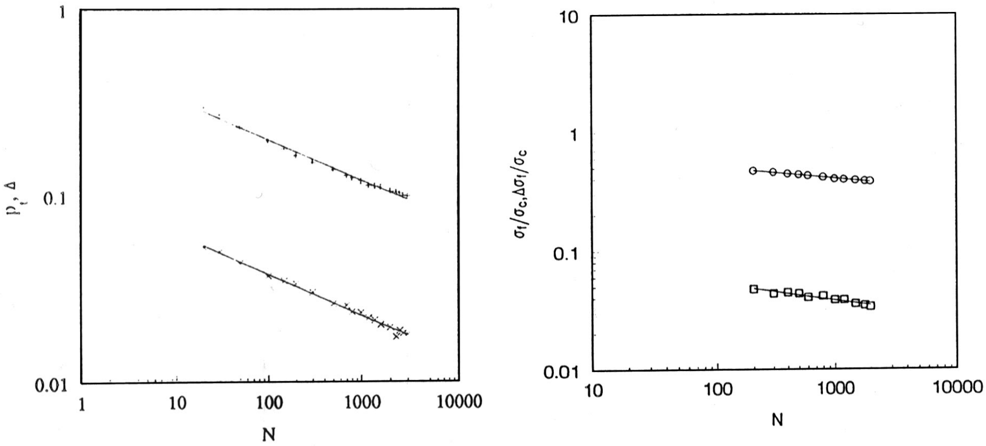

Figure 2: Size effect of transitional region (a) p_t and Δ, $\sigma_0/\sigma_c = 0.5$, $+ : \bar{p}_t$, $\times : \Delta$, (b) $\bar{\sigma_f}$ and $\Delta\sigma_f$, $p_0 = 0.2$, $\circ : \bar{\sigma_f}/\sigma_c$, $\Box : \sigma_f/\sigma_c$

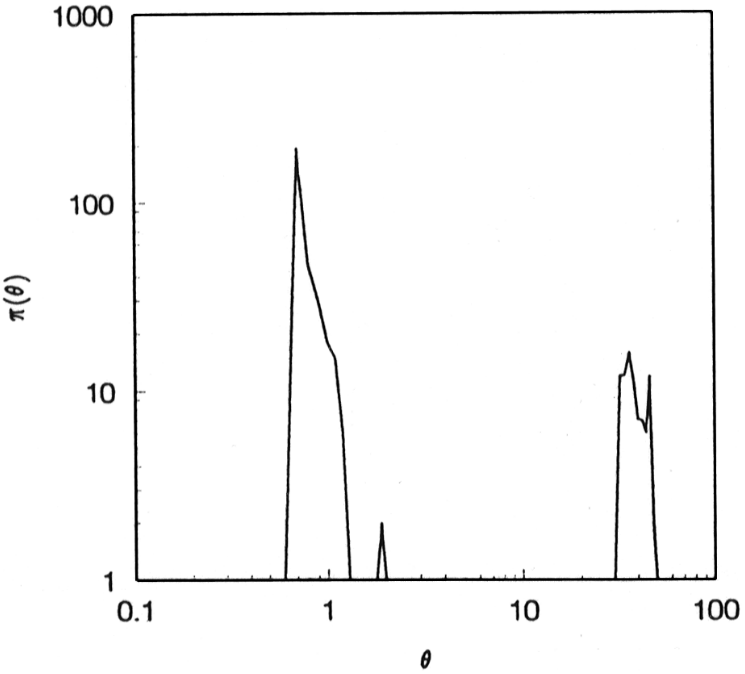

Figure 3: Ensemble probability distribution of maximum standard deviation of stress.