Rupture dynamics in 3-D

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Abstract

Using a 4th order finite difference program we have studied in some detail the propagation of seismic ruptures along a fault surface subject to a heterogeneous stress distribution and inhomogeneous frictional parameters. When prestress is uniform, rupture propagation is simple but presents some differences with the circular shear crack models of Kostrov. The most important are that rupture can only start from a finite initial patch (or asperity) and that rupture is not symmetric, so that the rupture front has an elliptical shape elongated in the inplane direction. If the initial stress is sufficiently high, the rupture front in the in-plane direction may jump ahead of the shear wave emitted from the initial stress release. In spite of these differences with Kostrov's model, all the relevant parameters scale with the fault length. When stress is heterogeneous rupture propagation changes completely and is controlled by local length scales determined by the initial stress distribution as well as by the distribution of rupture resistance. Thus short rise times (Heaton pulses), rupture arrest, stopping phases, etc reflect the length scales of the stress and strength distribution. Although we are only starting to explore the physics of rupture in a heterogeneous stress environment, we have identified a non-dimensional parameter κ that controls rupture. This parameter measures the ratio of local available strain energy to Fracture energy derived from the friction law. A bifurcation occurs when this parameter has values greater than a certain critical value that depends mildly on the geometry of the stress distribution on the fault. We expect all parameters inverted from seismic observations to scale with the shortest length scale inverted from near field observations.

Introduction

An essential requirement to study dynamic faulting is an accurate and robust method for the numerical modeling of seismic sources. In our recent work we have used a fourth-order formulation of the velocity-stress method (Madariaga, 1976; Olsen et al, 1995b) which was extended by Olsen et al (1997) and Madariaga and Olsen (1998) in order to study dynamic rupture propagation on a planar shear fault embedded in a heterogeneous elastic half space.

Rupture propagation on a major earthquake fault is controlled by the properties of the friction law between the sides of the fault. Friction controls the initiation, development of rupture and the healing of faults. Laboratory experiments at low slip rates were analyzed by Dieterich (1978), who proposed models of rate- and state-dependent friction, and by Ohnaka and Kuwahara (1990) who concluded that their

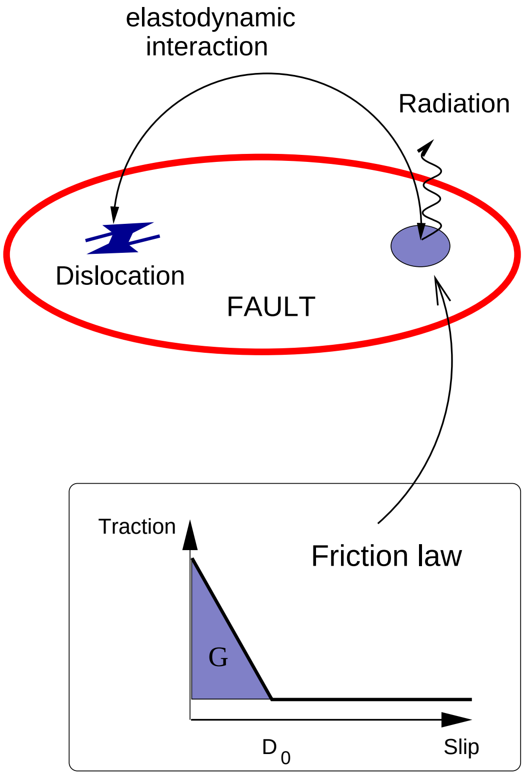

Figure 1: Geometry of a simple rupture propagating along a flat fault embedded in a 3D elastic medium. Rupture propagation is controlled by elastodynamic interactions and is damped by seismic radiation. The boundary conditions on the fault are dominated by friction. We use a simple slip weakening friction of peak stress T_u, slip weakening D_0 and energy release rate G

experiments could be explained with a simpler slip-weakening friction law. These two models to friction contain a finite length scale that controls the behavior of the rupture front. At high speeds the two friction laws are very similar.

Because of the equivalence of friction laws at high slip rates, we used a simple slip weakening friction law in which slip is zero until the total stress reaches a peak value (yield stress) that we denote with T_u. Once this stress has been reached, slip D increases and $T(D)$ decreases:

$$\begin{aligned} T(D) &= T_u\left(1-\frac{D}{D_0}\right) \qquad \text{for} \qquad D < D_0 \\ T(D) &= 0 \qquad\qquad\qquad \text{for} \qquad D > D_0, \end{aligned} \tag{1}$$

where D_0 is a characteristic slip distance. This friction law has been used in numerical simulations of rupture by Andrews (1976), Day (1982b) and many others.

Figure 1 shows the geometry of the fault model we study. The most important feature of the friction law is slip weakening that occurs near the rupture zone on a so-called break-down or slip-weakening zone just behind the rupture front. The propagation of the rupture front is completely controlled by the friction law and the distribution of initial stress on the fault. The numerical method we developed to solve this problem was to compute spontaneous rupture propagation for the Landers earthquake by Olsen et al. (1997).

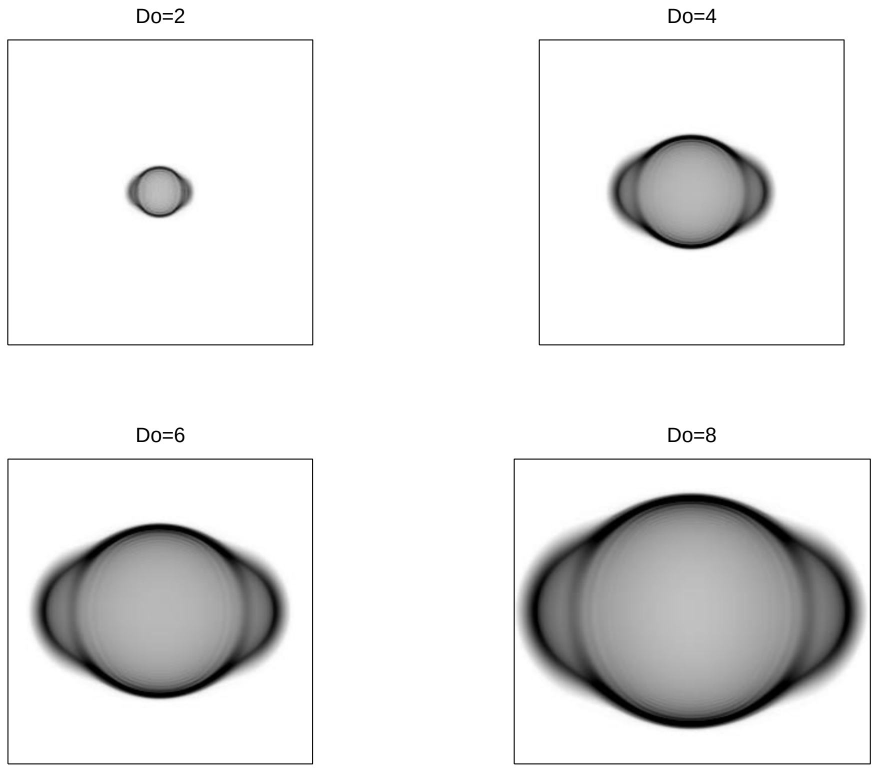

Figure 2: Rupture growth on a flat uniform fault embedded in a homogeneous elastic medium. Rupture starts from a finite initial asperity and then grows at subsonic speed in all space direction. After a while rupture along the inplane direction jumps at a speed that is higher than that of shear waves. The snapshot here shows the slip rate as a function of position on the fault at times greater than the jump to super shear speed along the in-plane direction (horizontal axis).

Spontaneous Rupture of a Localized Asperity

Madariaga et al (1998) studied the spontaneous rupture starting from a circular asperity of radius R that is ready to break (with stress T_u), and is surrounded by a fault surface at a constant but lower stress level ($T_e < T_u$). As many other authors have in he past we observed that for rupture to expand, stress must be high over a finite zone, sometimes called the minimum rupture patch. Once rupture has broken the asperity it will grow or stop depending on the values of the stress field inside (T_u) and outside (T_e), the shear modulus μ, the asperity radius R and the slip weakening constant D_0 of the friction law in (1).

Rupture resistance, or energy release rate for our slip weakening friction model (1) is $G = \frac{1}{2} T_u D_0$. In our model we assumed that G is uniform along the fault plane. This model is not unlike the classical circular model of Kostrov (1964). The main differences are that, as explained above, we initiate the rupture from a finite asperity and rupture velocity not constant but determined from the friction law.

After a very complete exploration of the parameter space for this model we found that there are two regimes in this problem that are controlled by a non-dimensional number

$$\lambda = \frac{T_e^2}{\mu T_u} \frac{R}{D_0} \tag{2}$$

where R is the radius of the minimum asperity size. A similar non-dimensional coefficient was first proposed by Andrews (1976) in a somewhat different context.

We can understand this number if we consider that λ is the ratio of available strain energy ΔW to energy release rate G defined above. Strain energy change in a zone of radius R is:

$$\Delta W = \frac{1}{2} < D > T_e = A T_e^2 / \mu R \tag{3}$$

where $< D >$ is the average slip on a fault of radius R and stress drop T_e. Thus $lambda \simeq \Delta W / G$.

An essential requirement for rupture to grow beyond the asperity is that $\lambda > \lambda_c$ where the critical value of λ for the circular asperity can be derived from the study by Madariaga et al (1998) who computed numerical the critical radius R_c for fixed T_u, $T_e xt$ and D_0. We find that

$$\lambda_c \simeq \frac{4}{3\pi} = 0.424 \tag{4}$$

λ_c defines a bifurcation of the problem as a function of parameter λ. For $\lambda < \lambda_c$ rupture does not grow beyond the initial asperity. For $\lambda > \lambda_c$ rupture grows indefinitely at increasing speed. This is a simple example of a pitch-fork bifurcation. But there is a complication: if the parameter $\lambda > 1.3\lambda_c$ the rupture front in the in-plane direction jumps ahead to speeds higher that the shear wave speed. The rupture front acquires a very nice elliptical shape with two "ears" elongated along the in-plane direction (see Figure 2).

Spontaneous Rupture on a Realistic Fault: The Landers 1992 earthquake

Olsen et al (1997) published a study of the Landers earthquake of July 1992 based on the kinematic model inverted by Wald and Heaton (1996). Careful study of rupture propagation in that model showed that rupture only propagates in regions of high stress that are sufficiently wide. The kinematic model was derived by Wald and Heaton (1996) from a combination of seismic and geodetic data that contains wavelengths down to about 3 or 4 km. This establishes the resolution length for the modeling of this earthquake. After some trial and error we fixed T_u at 12 MPa, which is close to the maximum value of the initial stress field shown in the first panel

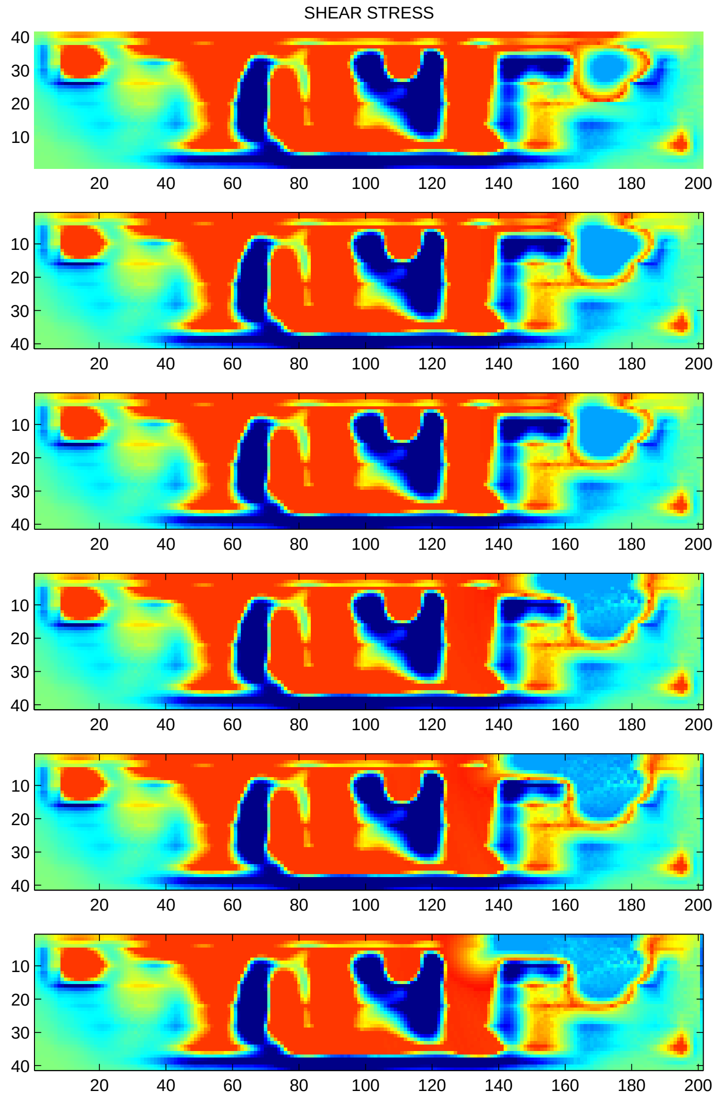

Figure 3: Snapshots of the shear stress field on the fault of the Landers 1992 earthquake. This is computed starting with the initial stress field shown in Figure 2.

in Figure 3. The only free parameter that remains to be determined is the initial rupture zone and the value of the slip weakening distance D_0.

The initial stress field shown in figure 3 shows that rupture can start from many points on the fault. We know that the Landers earthquake started from the small patch near the southern (right in the figure) end of the fault. So we took a circular zone satisfying condition (2) and then let it grow spontaneously. We find unexpectedly that it was extremely difficult to let rupture grow beyond the initial asperity. After several trials we found that we needed values of D_0 of about than 0.5 m for rupture to grow. With the value of $T_u = 12MPa$ this yields a rupture energy $G = 3MJm^{-2}$.

Then we looked at rupture conditions. The stress field is not very favorable to rupture initiation because the stress field surrounding the initial patch is rather weak. So, as shown in Figs 3 and 4 initial rupture takes a long time before reaching the vertical high stress zone to the North (left) of the initial patch. Once rupture enters into this zone it propagates downwards at high speeds because the loaded zone is relatively wide. Once it hits the bottom it stops because there is no lateral communication along the bottom of the fault.

In conclusion the initial stress field controls rupture propagation very closely. Rupture extends following a relatively clear pattern of infiltration. It goes into places where stress is high over large patches and avoids completely the zones where stresses are low. The conditions for growth of rupture can be understood in terms of a non-dimensional number like λ in (2).

Very Long Rectangular Asperity on a Flat Uniform Fault

In order to study the condition for rupture propagation in a heterogeneous initial stress field we turn to a very simple rectangular asperity model. We study a simple initial stress field that contains a long asperity of width W loaded with a longitudinal shear stress $T_e = 0.9 * T_u$. The asperity is surrounded by an infinite fault plane where stress is very low at only $0.1 * T_u$. At time $t = 0$ rupture is initiated by forcing rupture over a circular patch of radius $R > R_c$ where R_c is computed from (2) using $\lambda_c = 0.42$. Depending on the values of T_e and the width W, rupture either grows along the asperity or stops very rapidly. We are again in the presence of bifurcation with a critical value. We define a new non-dimensional number

$$\kappa = \frac{T_e^2}{\mu T_u} \frac{W}{D_0} \tag{5}$$

and verify numerically that ruptures stop for low values $\kappa < \kappa_c$ and grow indefinitely for $\kappa > \kappa_c$. Again for a certain value of $\kappa > 1.2\kappa_c$ ruptures grow initially at very high speeds and then jump to a speed higher than the shear wave velocity.

Figure 5 shows rupture propagation along the long rectangular asperity for three values of κ. On the top right we show a shear stress snapshot and on the right a slip rate snapshot at the same time for $\kappa < \kappa_c$. Rupture in this case starts near the asperity and then stops immediately. D_0 is to large compared to W. The second row shows stress and slip rate when κ is slightly super critical. A rupture propagates along the asperity at sub-shear speeds. As ruptures extends the rupture zone extends a little outside the the asperity leaving an elongated final fault shape. Finally, at the bottom of Figure 5 we show a stress an a slip rate snapshot when κ is about 1.2

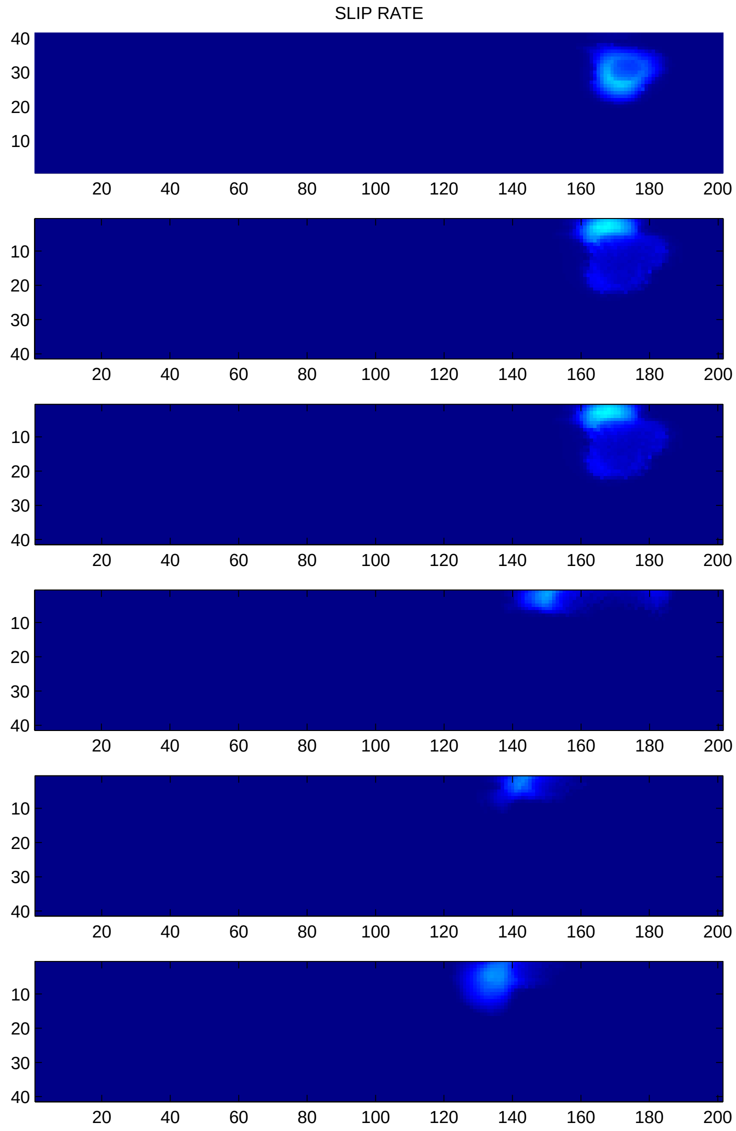

Figure 4: Snapshots of the slip rate field on the fault of the Landers 1992 earthquake. This is computed starting with the initial stress field shown in Figure 2.

times critical. Now the rupture front is running faster than the shear wave producing a wake that spreads somewhat into the lower prestress zone. Thus we are again in the presence of a bifurcation: it is not enough to start a rupture, the stress field has to be high enough to maintain rupture going.

We believe that this is the first time that conditions for rupture gow and arrest are studied in a 3D rupture model. Our results are entirely compatible with those of Huseini et al (1978) who studied conditions for rupture in a simple 2D fault.

Conclusions

We have studied carefully rupture growth in a simple flat fault embedded in a homogeneous elastic medium of rigidity μ. It emerges from our studies that rupture is controlled by a non-dimensional number $\kappa = \frac{T_e^2}{\mu T_u} \frac{L}{D_0}$ where L is a characteristic size of the stress field, for instance the patch (asperity) radius, T_e is a characteristic value stress load on this patch and $T_u \times D_0$ is a measure of energy release rate on the fault.

In our analysis so far we have assumed uniform rupture resistance (constant T_u and D_0. All the complexity comes from the heterogeneity of initial stress. As important as stress heterogeneity is probably the small scale geometry of faulting. Its integration in fault models is difficult because most of the observations of fault rupture are still limited to the range of frequencies of less than 1 Hz or about 3 Km wavelength. This is too coarse a resolution to observe effects of complex geometry other than major fault segmentation as in Landers of Kobe. High frequency seismic radiation is probably the only source of information about small scale geometry.

The implications of this simple number are wide ranging and require extensive tests and analysis of modeling. We are doing this for the case of the Landers earthquake.

Acknowledgments

The computations in this study were partly carried out on the SGI Origin 2000 at MRL, UCSB (NSF Grant CDA96-01954), partly on the Sparc20 server at ICS, UCSB, with support from NSF Grant EAR 96-28682) and the Southern California Earthquake Center (SCEC), USC 572726 through the NSF cooperative agreement EAR-8920136. R. Madariaga's work was supported by the Environment Program of the European Community under project SGME. Movies of the Landers earthquake simulation can be found on the World Wide Web at http://quake.crustal.ucsb.edu/~kbolsen.

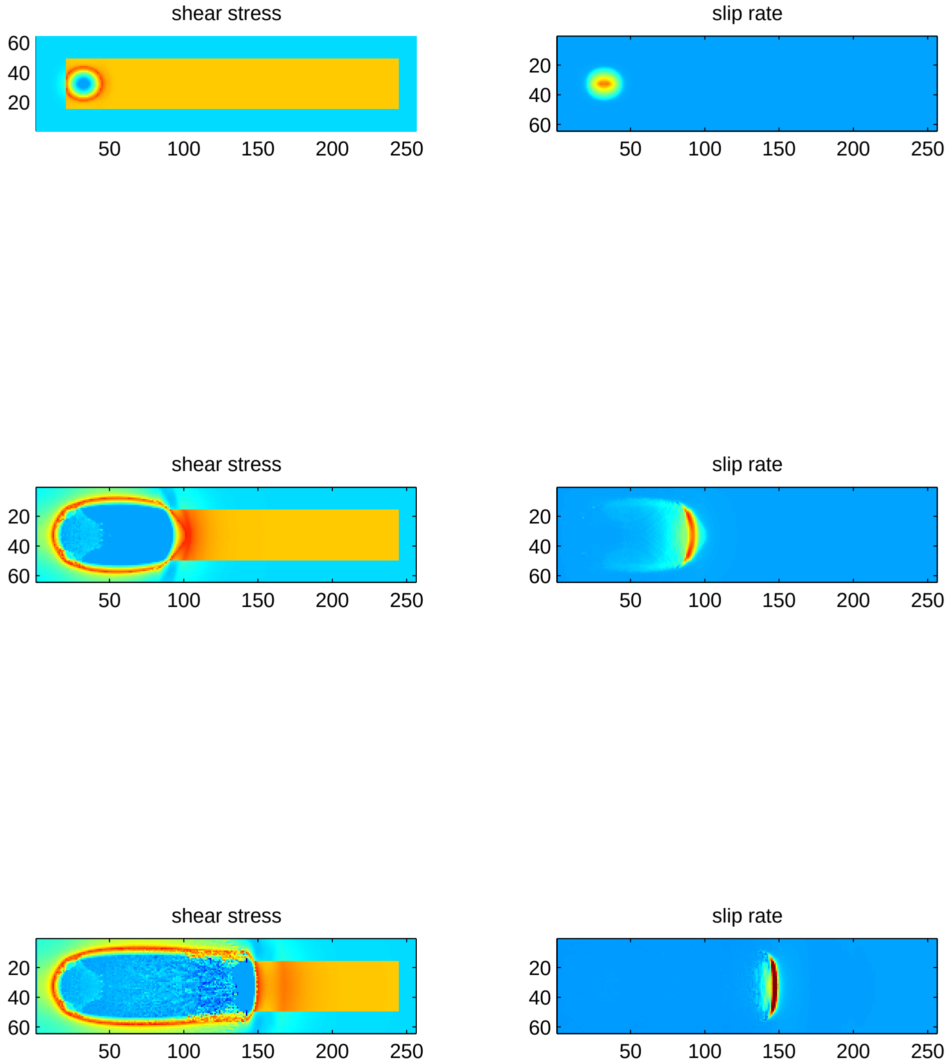

Figure 5: Snapshots of the shear stress and slip rate fields on a fault containing a very long and narrow asperity (yellow in the stress snapshots). The top panels show ruptures at sub-critical, slightly super critical and super critical values of the parameter κ.

References

[1] Andrews, J., 1976. *Rupture velocity of plane strain shear cracks,* J. Geophys. Res. **81**, 5679-5687. n

[2] Day, S., 1982, *Three-dimensional simulation of spontaneous rupture: the effect of non-uniform prestress,* Bull. Seis. Soc. Am. **72**, 1881-1902.

[3] Dieterich, J. (1978). Time-dependent friction and the mechanics of stick-slip, *Pageoph* **116**, 790-806.

[4] Heaton, T., 1990, *Evidence for and implications of self-healing pulses of slip in earthquake rupture,* Phys. Earth. Planet. Int. **64**, 1-20.

[5] Kostrov, B., 1964. *Self-similar problems of propagation of shear cracks,* J. Appl. Math. Mech. **28**, 1077-1087.

[6] Madariaga, R., 1976, *Dynamics of an expanding circular fault,* Bull. Seism. Soc. Am. **66**, 639-667.

[7] R. Madariaga, Olsen, K.B., and Archuleta, R.J., 1997, *3-D finite-difference simulation of a spontaneous rupture,* Seismol. Res. Lett **68**, 312.

[8] Ohnaka, M. and Y. Kuwahara, 1990, *Characteristic features of local breakdown near crack-tip in the transition zone from nucleation to dynamic rupture during stick-slip shear failure,* Tectonophysics **175**, 197-220.

[9] Olsen, K.B., J.C. Pechmann, and G.T. Schuster, 1995,*Simulation of 3-D elastic wave propagation in the Salt Lake Basin,* Bull. Seism. Soc. Am. **85**, 1688-1710.

[10] Olsen, K., Madariaga, R., and Archuleta, R., 1997, *Three dimensional dynamic simulation of the 1992 Landers earthquake,* Science **278**, 834-838.

[11] Wald, D. and Heaton, T., 1994, *Spatial and temporal distribution of slip for the 1992 Landers, California earthquake,* Bull. Seis. Soc. Am. **84**, 668-691.