Sensitivity of 3D rupture dynamics to fault geometry and friction parameters

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Abstract

We scale the various parameters defining a 3D fault model (i.e. characteristic distance and time of a state friction law, size and aspect ratio of the fault, medium impedance) and derive two dimensionless parameters governing the typical dynamics of the fault through single or multiple ruptures. The different faulting regimes are illustrated by a series of numerical simulations. As the parameters are varied the model crosses over from a regime which exhibits narrow, self-healing slip pulses, to one which exhibits broad, crack-like solutions that only heal in response to edge effects. In the crack-like regime we observe periodic systemwide events. For self-healing pulses, the system exhibits self-roughening which leads to dynamical complexity. The behavior also changes from periodicity or quasi-periodicity to more complex time sequences as the total fault size or the length to width ratio are increased. Our results are in good qualitative agreement with analogous results which we obtain for a one dimensional Burridge–Knopoff model, where the variations in the stiffness of the transverse spring are related to variations in the width of an equivalent two dimensional fault, and radiation effects are approximated by viscous dissipation.

The friction law

For our studies we define a rate and state friction law which incorporates a characteristic distance δ for slip weakening, a characteristic time τ for healing, and a velocity weakening steady state. We choose the simplest functional representation for these features: The friction is given by

$$F_0 = \begin{cases} [-\theta, \theta], & \dot{U} = 0 \\ \theta + \eta \dot{U}, & \dot{U} > 0 \end{cases} \quad , \tag{1}$$

where the state variable θ satisfies the evolution equation

$$\dot{\theta} = \frac{1-\theta}{\tau} - \frac{1}{\delta}\theta \dot{U} \quad . \tag{2}$$

Here U represents the relative local displacement of opposite sides of the fault (i.e., the slip) and the dot denotes a derivative with respect to time t. When the interface is stationary, $\dot{U} = 0$, the static friction threshold is set by the value of the single state variable, θ, which is confined to the unit interval: $0 < \theta < 1$.

The steady state friction applies when there is a uniform slip rate $\dot{U}_{\mathrm{ss}}$. In this regime the state variable exhibits velocity weakening:

$$\theta_{\mathrm{ss}} = \frac{1}{1 + \dot{U}_{\mathrm{ss}}\tau/\delta} \quad , \tag{3}$$

and when $\eta > 0$ the steady state friction $F_{\mathrm{ss}} = \theta_{\mathrm{ss}} + \eta\dot{U}_{\mathrm{ss}}$ exhibits a crossover from velocity weakening to velocity strengthening at a velocity $\dot{U}^{*}_{\mathrm{ss}} = \frac{\delta}{\tau}(\sqrt{\tau/\delta\eta} - 1)$.

Single rupture with homogeneous prestress

In order to evaluate the consequences of varying the friction parameters we have simulated the 25 combinations of $\delta = [1.8, 1.85, 1.9, 1.95]$ and $\tau = [45, 46.25, 47.5, 48.75, 50]$, in a model of 2D fault embedded in a 3D acoustic medium [*Madariaga et al.*, 1996]. A quick overview of the results for the different (δ, τ) pairs is illustrated in Figure 1, where a series of snapshots representing the velocity of the actively slipping patch on the fault after 140 dimensionless units of time is shown. Darker regions in gray scale represents faster slip rates.

Our numerical results suggest that at least for the range of parameters we have chosen, the pulse width and rise time vary more systematically with the ratio δ/τ than with independent variations of either parameter. This ratio has the dimensions of a velocity, and appears in (3) as the characteristic velocity of the steady state friction. In its stationary approximation (3), our friction law resembles that used by *Cochard and Madariaga* [1996] and, similarly, its ability to generate pulses is governed by the level of rate-weakening (controlled by δ/τ).

Scaling the friction parameters

As seen in the former section, the self-healing ability of the propagating rupture pulse is essentially controlled by the characteristic velocity δ/τ appearing in the stationary friction expression (3).

Let a fracture propagate inside a homogeneous medium of stiffness μ and wave velocity β. If the dynamic stress drop (i.e., the difference between the initial stress and the friction during dynamic sliding) associated with the fracture occurrence is of the order of $\Delta\sigma$, then dimensional arguments tell us that the sliding velocity ought to scale as $\Delta\dot{u} \approx \Delta\sigma\beta/\mu$. This simple result is confirmed by all the analytical solutions available (see for example *Dahlen* [1974], for an elliptical crack with prescribed propagation velocity). Under the assumption that the dynamic sliding friction is low, $\Delta\sigma$ can be equated to the prestress, yielding an a priori estimation of the characteristic slip rate.

Therefore we can scale these two velocities with each other and argue that the ability to generate premature healing is controlled by the dimensionless parameter $H = \Delta\sigma\beta\tau/(\mu\delta)$. In other words, if the typical sliding rate is high in comparison to the characteristic rate δ/τ, the dynamic sliding friction will tend to be low (because

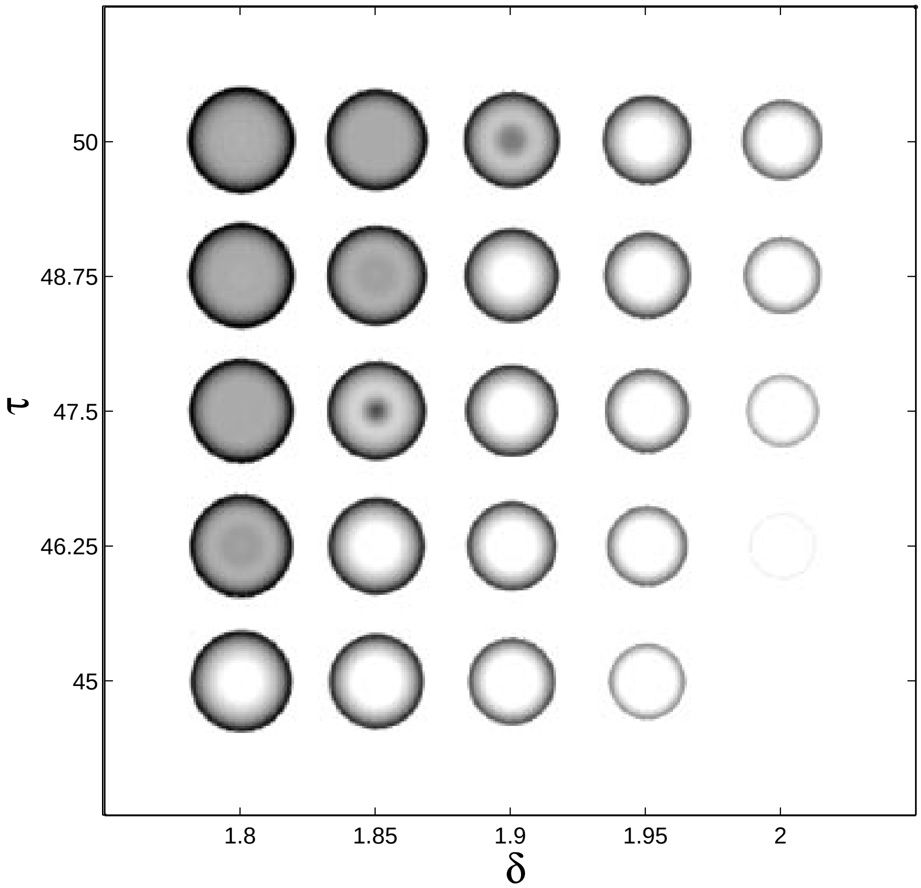

Figure 1: A quick overview of the results for various (δ, τ) couples is presented as a series of snapshots. Each circular pattern represent the actively slipping area of one fault after 140 dimensionless units of time, centered on the (δ, τ) value used to produce it. Variations in both radius and pulse width are observed. However, the patterns are very similar along a diagonal of given slope, showing that the outcome is mainly sensitive to the δ/τ ratio.

of rate-weakening) and no self-healing will occur. On the other hand, the rupture will stop very rapidly whenever the slip is relatively slow. However, note that the stress drop $\Delta\sigma$ is not precisely known a-priori, because it depends on the level of dynamic friction during sliding; moreover, the velocity at which healing occurs is not precisely δ/τ, although it does depend on δ/τ. Hence, only approximate predictions can be issued based on the actual value of H in the absence of further computations. A more specific solution for the cross-over between the pulse-like and the crack-like solutions can be found in *Zheng and Rice* [1998], under the approximation of a purely rate-weakening friction.

Examples of recurrent ruptures

In this section we consider recurrent ruptures in the continuum model. We begin with a randomly generated spatially inhomogeneous initial stress configuration, and simulate a sequence of large events by reloading the fault once each rupture is terminated and the radiated waves have propagated into the absorbing boundaries. The final state of stress on the fault is used as the initial condition for the next rupture, after increasing the load sufficiently to induce nucleation of another large event.

We first observe a transient phase, consisting of a series of quasi-periodic ruptures extending throughout entire fault, which ultimately evolves to a statistically stationary state where the stress distribution is no longer dependent on the initial conditions. It is in this later phase that we focus on to determine how the friction parameters and system geometry affect the level of dynamic complexity. In this section we distinguish between three different regimes– strict periodicity, aperiodic systemwide events, dynamical complexity– that can develop in a system of fixed size, depending on the characteristic velocity δ/τ. The latter regimes are associated with different levels and variations of heterogeneity in the stress field.

(1) Periodicity: For $\delta/\tau \approx 0.004$ and below we observe periodic large events, which generate smooth, repeating stress fields. Beginning from the random initial state, after a few cycles of large events the system settles into a periodic state. The particular features of the periodic large event (e.g. the nucleation site) depend on the initial state, though the general property of periodicity does not. The periodic state is associated with broad crack-like pulses, in which healing occurs in response to boundary effects.

(2) Aperiodic Systemwide Events: For intermediate values of δ/τ: $0.004 < \delta/\tau < 0.04$ we observe aperiodic large events which span the system, generating smooth but variable stress fields. The nucleation sites and dynamical evolution of the individual large events vary. As in the case of periodicity, the large events consist of crack-like pulses, which heal in response to boundary effects.

(3) Dynamical Complexity: For $\delta/\tau > .04$ we observe a broad distribution of sizes of events and a highly heterogeneous and variable stress field. It is in this regime that we observe narrow, self-healing slip pulses as shown in the previous section. For larger values of the slip weakening length δ it is difficult for the pulse to propagate into fresh territories, while for smaller values of τ (larger values of the healing time δ/τ) healing takes place more rapidly. When the stress field is (self-consistently) heterogeneous, the narrow pulses are often arrested before reaching the boundary. This results in the formation of singular features in the stress configuration at the perimeter of slip. In the final configuration this sharp feature separates the patch which actively slipped where the overall stress is somewhat reduced, and the patch which did not slip, which in which the stress is somewhat increased due to long range elastic effects. The narrow peaks of high stress typically comprise the nucleation sites of subsequent ruptures, and the stress heterogeneities sustain the broad distribution of events.

In Figure 2, we illustrate a sequence of scatter plots for the area of slip vs. the duration of rupture for three different values of δ/τ corresponding to the three different regimes.

Figure (a) illustrates the periodic regime ($\delta/\tau = 0.004$) in which all of the events are coincident after four initial transient ruptures. Figure (b) illustrates the aperiodic

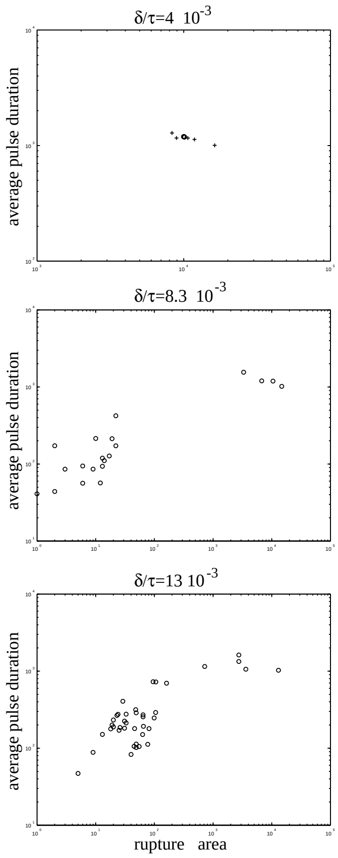

Figure 2: Three different seismic regimes obtained on the same fault by varying the value of δ/τ in the friction. Every circle or cross corresponds to an individual rupture event on the fault during a recurrence cycle, in this particular phase diagram representing pulse duration versus rupture area. The rupture area is defined as the total area fractured during a given event. The average pulse duration is the duration of active slip at a given point of the fault, during a given event, averaged over the whole fracture area. The top plot, for a relatively low δ/τ, yields periodic behavior. The first 5 events, represented as crosses, converge to a stable limit cycle, after what the remaining 15 events, represented by circles, are virtually identical and overlap in the phase diagram. In the middle plot, the periodicity is lost by increasing δ/τ, but there are roughly two population of events: fault wide characteristic events which area clusters around 10^4, and smaller events representing nucleations that did not propagate further than the initial rupture patch. Thus we do not get complexity but rather a bi-modal distribution in this case. However, in the bottom plot, representing the simulation with the largest δ/τ, we start to see a broader distribution of sizes. A rupture in the slope indicates the size after which the rupture propagates in self-healing pulses (the duration becomes constant and does not grow with the area of fracture any more) as opposed to crack at smaller sizes.

case. Here the large events for a more diffuse population on the graph. While they all span the system (thus have equal area), their durations vary due variations in the position of the nucleation site relative to the boundaries. In this case there is also a population of smaller events in which nucleation criterion fails to produce a propagating rupture. Instead, the slip amplitude rapidly decays after the initial nucleation. Finally, Figure (c) illustrates the regime in which we observe dynamical complexity. In this regime, we have a broad distribution of the sizes of events of variable area and duration. The system develops well formed propagating, self-healing pulses which terminate before reaching the boundary.

Finally we comment on the effects of varying the overall size and aspect ratio of the system. The reduction and expansion of size of the fault should lead to limiting behaviors. For example, for a fixed value of δ/τ which exhibits complexity on a larger system, reducing the system size will reduce the probability that slip pulses will encounter stress inhomogeneities which are sufficiently large to terminate the rupture, which in turn increases the likelihood of systemwide events. Indeed, we observe a crossover from regime (3) to (2) as the fault size is reduced. However, we have not seen the onset of periodicity by reducing the fault size further. Instead, we run into the unphysical situation in which the size of the fault is close to the characteristic size associated with our nucleation criterion first. For this reason it would be preferable to study size effects by beginning with a periodic state and looking for crossovers to different regimes as the size of the fault is *increased*. However, the extent to which we can increase the system size is sufficiently limited by computational resources, that we are unable to obtain conclusive results. On the other hand, we do observe that above a critical value of H (which scales with system size) the average pulse width becomes independent of the fault size. Alternatively, *Rice* [1993] discussed the onset of complexity when the size of an elementary sub-fault element became large with respect to a characteristic frictional distance.

Similarly, variations of the aspect ratio of the fault can lead to a crossover from regime (2) aperiodic systemwide events to (3) dynamical complexity. As noted above, reduction of the linear size of a square fault eventually leads to slip pulses where the width depends on the system size. In this regime, when the length L and width W of the fault are different, it is the shortest length scale (here assumed to be W) which controls the width of the slip pulse. Then, when L is increased for a fixed value of W we observe a crossover to dynamical complexity.

In the following section we discuss a number of simulations where the ratio L/W takes several values in the interval $[2.2; 12]]$. On the side of the spectrum closest to the aspect ratio of 1, for $W/L = 2.2$, the fault is quasi - periodic in the sense that all events that do propagate past the nucleation area systematically reach the entire fault, yielding characteristic size events at characteristic time intervals, with minor variations. This seems to continue even after the initial transient is suppressed (i.e. after about 10 ruptures that are still affected by the arbitrary initial condition).

Interestingly, when the aspect ratio is reduced, there is only an initial transient interval during which all propagating ruptures reach the entire fault but then some intermediate size events start to appear.

When the aspect ratio is reduced even more, only intermediate size events remain after the initial transient and no rupture reaches the whole fault any more (e.g., $L/W = 12$ in Fig. 3). The event size distribution becomes wider and it is not limited at the high end by the finite fault length.

Only the last third of the time axis, away from the initial transient, is significant in terms of the self-generated stationary regime of the fault, where the difference between the regimes is most clear.

Changes in the friction parameters and system size have a qualitatively similar effect on the complexity of the BK model and the continuum system, provided that a form of radiation damping is simulated in the BK model, for example by adding a viscous term to the friction. Larger values of τ, smaller values of δ, and reduction of the system size lead to more regularity in the dynamics. However, unlike our continuum simulations, in the BK model for a given set of friction parameters we have always been able to find system sizes which are large enough to generate dynamical complexity. The clear exception to this arises in the case where the steady state friction is velocity strengthening (i.e. $\beta > \tau/\delta$), as pointed out in the context of a different friction law by Shaw. Because computational resources limit the range of system sizes we can consider for the continuum, it is impossible to conclude whether

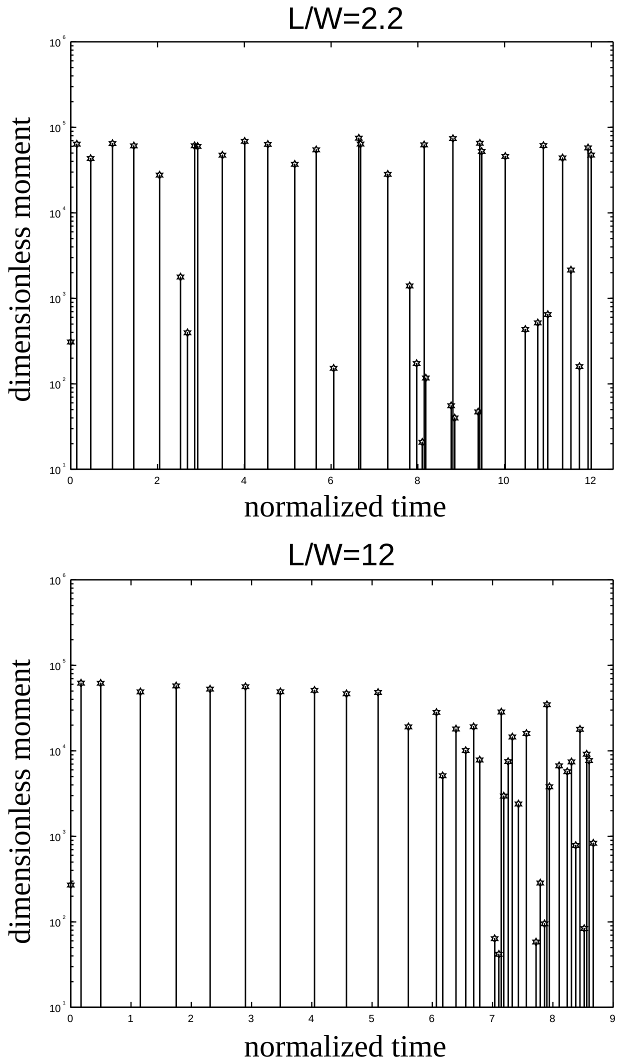

Figure 3: The two time sequences above illustrate the qualitative change in the fault recurrence dynamics when the aspect ratio L/W (length versus width of a rectangular fault) is changed. The ten first events in both catalogs should be discarded, they only are representative of a transient initial regime where the fault is still influenced by an arbitrary imposed initial stress. The subsequent ruptures are part of a self-generated stationary regime on the fault which is our focus of interest. In this later part of the time sequences, a fundamental difference between the two models with different length to width ratios is that in the case where $L/W = 2.2$ (top), the fault persistently undergoes ruptures that span its entire area, whereas none such rupture is observed in the case where $L/W = 12$ after the initial transient has terminated. As a consequence, intermediate-size events with various moments and time intervals are generated in the long narrow fault, because ruptures propagate in pulses that die out before reaching the unbreakable extremity of the fault. In the short fault $L/W = 2.2$, those intermediate events are being cut off by the fault extremity before they can spontaneously die-out, thus generating characteristic size events and characteristic time intervals; the smaller ruptures in this case are nucleations that die out immediately and form asperity-like ruptures that won't propagate.

complexity would ultimately arise in larger systems in the continuum case.

Conclusions

In summary, we have investigated properties of the rate and state law introduced in Section II in the context of a three dimensional continuum model of faulting and a BK model. In the continuum we observe regimes consisting of crack-like pulses which heal in response to edge effects, as well all narrow, self-healing pulses, which are independent of boundary conditions. For repeated ruptures, crack-like pulses coincide with periodic or quasi-periodic large events that span the system. The onset of dynamical complexity coincides with the crossover to self-healing. It is also in this regime that we begin to encounter problems with numerical resolution of sharp features in the stress configurations, although long wavelength properties are well resolved, and the behavior is qualitatively insensitive to grid effects. The system size and aspect ratio of the fault play a role in determining the crossover, though it is difficult to quantify due to limitations imposed by computational resources.

The value of the dimensionless quantity $H = \beta\Delta\sigma\tau/\mu\delta$ sets the crossover between different dynamical regimes. Here H is the ratio between the characteristic slip rate $\Delta\sigma\beta/\mu$, which is associated with a stress drop $\Delta\sigma$ in a medium of stiffness μ which propagates waves at velocity β and the characteristic velocity δ/τ associated with the friction. The transition from periodic, to quasi-periodic, to complex dynamics takes place in the numerical simulations as H is reduced, which leads to the development of self-healing pulses. Self-healing introduces strong heterogeneities in the stress field which typically act as initiation or termination locations for subsequent ruptures.

For large values of H, self healing pulses do not form in the middle of the fault surface, but only once that the rupture has reached the unbreakable boundaries of the fault, and the pulse length is controlled by the smaller dimension of the fault (i.e., the width W). We have observed cases for Then, in the case of a fault surface of aspect ratio close to 1, the fault settles in a periodic cycle of recurrent ruptures. However, if the dimension of the fault is increased, keeping the same aspect ratio and the same parameters, the regime becomes more complex and periodicity is lost. Alternatively, in the case of a long and narrow fault, i.e., $L/W < 10$, the pulse length $l_p \approx W$ is quite smaller than the maximum fault length L, and a degree of complexity is observed even for large H values. Indeed, after a transient regime affected by the initial conditions, the fault settles into a recurrence pattern where no rupture reaches the entire length of the fault, and a wide spectrum of event sizes is produced as opposed to the periodic cycle of fault-wide events observed at lower values of W/L.

These results for a fault embedded in a 3D continuum are compared to the dynamics of a Burridge-Knopoff lattice. It is argued that the parameter μ/W in the continuum can be interpreted as the stiffness k of the transverse spring in a BK lattice, where both quantities scaled by the total length L tell us something about the degree of complexity obtained.

As far as pulse-like behavior is concerned, the dependence of pulse length on δ/τ is also observed in th BK models, provided that a viscosity is added in order to simulate parametrically the effect of energy loss by radiation.

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