Linked stress release model for spatio-temporal seismicity

Chunsheng Lu and David Vere-Jones

Institute of Statistics and Operations Research, School of Mathematical and Computing Sciences, Victoria University of Wellington, Wellington, New Zealand (e-mail: cslu@mcs.vuw.ac.nz; dvj@mcs.vuw.ac.nz, phone: +64-4 472 1000, fax: +64-4 495 5045).

Abstract

A linked stress release model, incorporating stress transfer and spatial interaction between different seismic regions, is proposed. The model is applied to spatio-temporal seismicity analysis of historical earthquake catalogues and synthetic ones generated by physical models. The results highlight the triggering mechanism of earthquake occurrence and the evidence that the crust may lie in a near-critical or self-organised critical state due to the long-range interaction. The spatio-temporal complexity of seismicity is closely related to both the nonlinear dynamics and heterogeneities in a seismic region.

Introduction

Based on field observations of the 1906 San Francisco earthquake, Reid proposed an elastic rebound theory of earthquake origins [1]. According to the theory, deformation and stress in a seismically active region accumulate due to relative movement of tectonic plates. When the stress exceeds a certain threshold, for example, the strength of rock media, an earthquake occurs and the accumulated strain energy is released in the form of seismic waves. Although this model and its modifications (so-called time and slip-predictable models by fixing only strength or residual stress in the model) have been widely used in long-term prediction, real sequences of large earthquakes are fundamentally more complicated. In particular, the elastic rebound theory suggests that a large earthquake should be followed by a period of quiescence, whereas in reality a strong earthquake can be followed by a period of activation and sometimes another earthquake of comparable magnitude.

The plate tectonics indicates that the lithosphere of the solid Earth can be regarded as a hierarchy of blocks with different length scales, from major tectonic plates to grains of rocks, which are interlinked by weak boundary zones or interfaces [2]. Current researches on the nonlinear dynamics of earthquake occurrence have shown that the crust may lie in a critically stable state and that earthquakes are consistent with a typical self-organised criticality (SOC) phenomenon [3, 4], in which the spatio-temporal fingerprints are power-laws of the magnitude-frequency relationship and fractal patterns of epicenters or hypocenters distributions. One of the possible mechanisms exhibiting SOC is due to the competition between local strengthening and weakening through short or long-range interactions, i.e, the triggering mechanism of earthquake occurrence.

In this paper, we will extend the stress release model, a stochastic version of the elastic rebound theory suggested by Vere-Jones [5], to consider the stress interaction among subregions and apply this new model to both the historical and synthetic earthquake catalogues. The results mentioned here are part of our Marsden Fund project "Nonlinear Modelling of Fracture Mechanisms" whose overall aim is to develop improved physical and statistical models for rock fracture and earthquake occurrence.

Linked stress release model

In the univariate stress release model, the regional stress level $X(t)$ increases deterministically between two earthquakes and releases stochastically as a scalar Markov process. The evolution of stress versus time is assumed to follow the equation

$$X(t) = X(0) + \rho t - S(t) \quad , \tag{1}$$

where $X(0)$ is the initial stress level, ρ is the constant loading rate from external tectonic forces, and $S(t) = \sum_{t_i<t} S_i$, where t_i, S_i are the origin time and stress release associated with the i-th earthquake [6, 7].

Obviously, stress transfer and interaction can not be considered in this simple stress release model. To take into account interactions between different subregions, the evolution of stress $X_i(t)$ in the i-th subregion versus time can be rewritten as

$$X(t,i) = X(0,i) + \rho_i t - \sum_j \theta_{ij} S(t,j) \quad , \tag{2}$$

where $S(t,j)$ is the accumulated stress release in the subregion j over the period $(0,t)$, and the coefficient θ_{ij} measures the fixed proportion of stress drop, initiated in the subregion j, which is transferred to the subregion i. Here, θ_{ij} may be positive or negative, resulting in damping or excitation respectively. It is convenient, if ignoring aftershocks, to set $\theta_{ii} = 1$ for all i. We shall call this new version a *linked stress release model* [8]. If $\theta_{ij} = 0$ for all $i \neq j$, the model is reduced to an independent combination of simple forms as in Eq. (1).

The value of stress released during an earthquake can be estimated from its magnitude. The magnitude M is proportional to the logarithm of the seismic energy E released during an earthquake according to the relation $M = \frac{2}{3}\log_{10} E + \text{const.}$ For simplicity, the stress drop during an earthquake is supposed proportional to the square root of the released energy, i.e., $S \propto E^{1/2}$. Then, we have the formula

$$S = 10^{0.75(M-M_0)} \quad , \tag{3}$$

where M_0 is the normalised magnitude.

The probability intensity of an earthquake occurrence is controlled by a risk function $\Psi(x)$. The simplest choice of $\Psi(x)$ is taken as an exponential function $\Psi(x) = \exp(\mu + \nu x)$, where μ and ν are constants and ν indicates the sensitivity to risk. This is a compromise between time-predictable and purely random (Poisson) processes. Here, the key for statistical analysis is that the data in either historical or synthetic earthquake catalogues can be treated as a point process in time-stress space with the conditional intensity function

$$\lambda(t,i) = \exp\left\{a_i + \nu_i\left[\rho_i t - \sum_j \theta_{ij} S(t,j)\right]\right\} \quad , \tag{4}$$

where $\alpha_i(= \mu_i + \nu_i X_i(0))$, ν_i, ρ_i and θ_{ij} are the parameters to be fitted. We choose to parameterise the intensity in this form because it is more amenable to physical intuition. The seemingly excess parameters are in response to fixing $\theta_{ii} = 1$ for all i. A simpler parameterisation can be recovered by setting $b_i = \nu_i\rho_i$, $c_{ij} = \theta_{ij}/\rho_i$. Estimates of the parameters are found by maximising the log-likelihood

$$\log L = \sum_i \left(\sum_j \log\lambda(t_j,i) - \int_0^T \lambda(t,i)dt\right) \quad , \tag{5}$$

where the observation interval $(0,T)$ contains events at times $0 < t_1 < t_2 < \ldots < t_N < T$. This can be done numerically by using the routines in the statistical seismology library (SSLib) [9].

With discretion in our choice of degree of interaction (and in the subregions if necessary), we have a number of possible models. The choice among these will be based on the Akaike information criterion (AIC), which is defined as

$$\mathrm{AIC} = -2\log\hat{L} + 2k \quad , \tag{6}$$

where $\log\hat{L}$ is the maximum likelihood for a given model and k is the number of parameters to be fitted in the model. This represents a rough way of compensating for the effect of adding parameters, and is a useful heuristic measure of the relative effectiveness of different models, in avoiding overfitting. For example, the simple stress release model ($\theta_{ij} = 0$ for $i \neq j$ in Eq. (4)) with three parameters as against the Poisson model with only one (α_i) or the Poisson with exponential trend with two (α_i, $\beta_i = \rho_i\nu_i$), must demonstrate a significantly better fit to justify the additional parameters.

Using different combinations of the parameters in Eq. (4), we can examine different stress interaction mechanisms. If we suppose all the parameters $\theta_{ij} \neq 0$, we are allowing long-range interaction. On the other hand, if we only let $\theta_{ij} \neq 0$ for neighbouring regions i and j, short-range interactions will predominate. The best model is that for which AIC has the smallest value.

The different versions of this model have been implemented to both the historical earthquake catalogues (China, Japan, New Zealand etc.) and synthetic catalogues generated from several physical models.

Application: two typical examples

Japanese historical earthquake catalogues

As an active island arc structure, Japan seems unlikely to consist of a few relatively well-defined seismic components, but rather of a closely interacting, highly fragmented ensemble within which a few larger units are embedded. The definition of appropriate subregions, which has to satisfy both geophysical and statistical requirements, is not in any case trivial, and the regions provided in the following analysis are not claimed to be optimal.

One way that might be used to define subregions is by the application of some clustering algorithm, with boundaries drawn equidistant between neighbouring clusters. This should recognise implicitly the geophysical structure of the region. An additional consideration is that subregions must include sufficient observations to allow the numerical parameter fitting procedure to converge. A related problem is the completeness and reliability of historical records. The best historical records are probably those for the Tokyo-Kamakura region in Japan, and the catalogue should be reasonably complete for events since 1400 with magnitude $M \geq 6.5$. We shall follow Zheng and Vere-Jones [7] in our identification of appropriate subregions, as shown in Fig. 1.

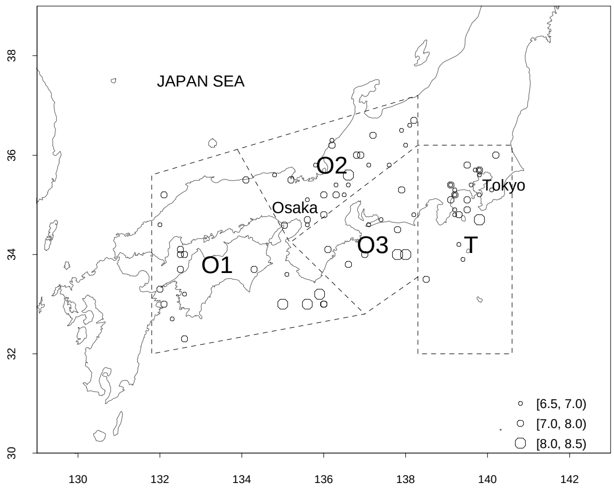

Figure 1: The epicenter distribution of major earthquakes with magnitude $M \geq 6.5$ and subregions of the Japan historical earthquakes. The Osaka region is divided into three subregions (O1, O2 and O3) and Tokyo is viewed as one subregion (T). We number the regions 1=O1, 2=O2, 3=O3, and 4=T.

We shall consider two factors in forming our model. Firstly, we have the choice of local or long-range interactions, the difference being that in the former case $\theta_{14} = \theta_{24} = \theta_{41} = \theta_{42} = 0$ (see Figure 1). There is also the opportunity to reduce the parameters $\{\rho_i\}$ to one constant ρ, representing a constant tectonic loading force across the subregions. Hence we have 4 possible models; long-range/non-constant, local/non-constant, long-range/constant, and local/constant, with 24, 20, 21 and 17 parameters respectively.

The best fit (with a significantly smaller AIC value) is obtained by letting all the interacting parameters $\theta_{ij} \neq 0$ and setting $\rho_i = \rho$ for all i. In this case we have 21 parameters, with a calculated AIC = 654.124. Due to the complicated makeup of the region, with each subregion being an ensemble of many closely interacting fragments, it is obvious that loading would be a complex aggregation of many elements. Secondly, loading imbalances could be equalised on a scale below that of the large events considered in the study, or through aseismic slip. Thus a constant loading rate could best explain the portion of the tectonic loading that acts to produce large earthquakes.

Carefully checking the parameters in Table 1, we find that earthquakes occurring in subregions O3 and T will increase the stress (risk) level in subregions O1 and O2, and *vice versa* since the related parameters are negative (except θ_{41} is positive). This indicates that the direction of the maximum compression stress is mainly in the S–E direction. Although it is difficult to obtain a clear indication for each subregion since the data are too small to study in detail, the general characteristic seems to be in agreement with existing results in the Japanese Islands found from the two surveys in 1883 to 1909 and 1948 to 1967 [10].

Subregion i	α_i	ν_i	ρ_i	θ_{i1}	θ_{i2}	θ_{i3}	θ_{i4}
1 (O1)	-7.260	0.0053	1.599	1.000	1.720	-1.805	-1.388
2 (O2)	-8.019	0.0100	1.599	0.276	1.000	-0.721	-0.323
3 (O3)	-5.350	0.0083	1.599	-0.012	-1.628	1.000	1.401
4 (T)	-6.128	0.0107	1.599	0.149	-0.939	0.308	1.000

Table 1: The fitted parameters using the (long-range, ρ = const.) linked stress release model, where the number of parameters $k = 21$ and the maximum log-likelihood $\log \hat{L} = -306.062$.

Moreover, the long-range (21 parameters) model is significantly better in AIC terms than the short range (17 parameters) model. Thus there is evidence to suggest long-range interaction of elastic stress between different subregions. Hence larger earthquakes in one part of a region may trigger other earthquakes in other quite distant parts of the region.

Synthetic catalogues generated by a block lattice model

In the model that we will describe, the brittle crust is simply represented by a square lattice of elastic blocks. Two neighbouring blocks interconnect via a combination of a normal spring, a tangential spring and a friction contact (fault). The total strain energy E_{total} in the system can be calculated by

$$E_{total} = \sum_{<i,j>} E_{ij} \quad , \tag{7}$$

where the sum is over all pairs of the nearest neighbouring blocks and E_{ij} is the strain energy concentrated between two contact blocks i and j.

The mechanical properties of a fault between two contact blocks are simulated by the Burridge-Knopoff stick-slip model [11], that is, a slip event or an earthquake triggering condition is controlled by the Coulomb criterion: $\tau \geq \mu\sigma + s_t$, where τ and σ are the incremental shear and normal stresses on the fault plane respectively, μ is a static coefficient of friction, and s_t is the cohesion strength on the plane. Therefore, for two contact blocks i and j, if the following condition is met

$$k_t|y_i - y_j| \geq -\mu k_n(x_i - x_j) + s_t \quad , \tag{8}$$

a slip occurs. In the system as a whole, a slip may trigger a cascade of slips like an avalanche process. Here, a series of slip events are regarded as an earthquake, and the whole dissipated energy during this process is equal to the energy released during an earthquake. The tectonic and lithostatic stress is simulated by an isotropic compression stress, and its value can be approximately determined by the relative deformation of the brittle crust.

Further details of the model ware given in Ref. [12]. The dynamics of the model at each time step is summarised as follows:

(1) A small slip displacement is added along the upper and lower sides, and the whole system is relaxed by minimising the total energy.

(2) Check the slip condition in Eq. (8). If the condition is satisfied between two contact blocks, let the tangential springs break and relax the whole system.

(3) Repeat the procedure (2) until there is no slip condition satisfied. Then reinstate the tangential spring and reset new values for the cohesion strength of the blocks slipped. Return to the procedure (1).

The simulation results showed that the coherent system naturally evolves into a SOC state. The magnitude-frequency distribution of synthetic earthquakes approximates the GR law with the $b = 1$ in agreement with the observational results [13], and the fractal feature of fault patterns are also reproduced. Here, the synthetic catalogues have some obvious advantages over real earthquake catalogues. They are free of observational errors or incompleteness and can be made as long as needed to obtain good statistical samples. In addition, the underlying physical processes are completely known [14].

In the present model, the released energy during an earthquake is obtained by a global minimisation of total energy. Hence, the stress redistribution is long-range rather than short-range in common automata models. As discussed above, this can be checked by the linked stress release model. The analytical results obtained testify the existence of a spatial long-range correlation and highlight the affective factors on the spatio-temporal complexity of seismicity.

Conclusion

A linked stress release model, incorporating stress transfer and spatial interaction, is proposed in this paper. This model fits both the historical and synthetic earthquake data better than a collection of independent simple models. The results provide a possible hint that the crust may lie in a near-critical state due to a long-range interaction of elastic stress, and provide a paradigm whereby spatio-temporal complexity of seismicity is related to both the dynamics and heterogeneities in a region. An earthquake occurring in a region could trigger another earthquake distant from it in the region. Despite the crudity of the model in physical terms, it has the advantage of fitting simple physical ideas into a stochastic framework, which allows it to be objectively fitted and tested on real data. Using the fitted parameters, we can forecast the long-term risk in a region.

Acknowledgments

This work was supported by the Marsden Fund administered by the Royal Society of New Zealand. We acknowledge many valuable discussions with D. Harte, M. Bebbington, and Y. Shi. C. Lu is also grateful to H. Takayasu for helpful suggestions and the support by Japan Society for the Promotion of Science Invitation Fellowship while he was a research fellow at Tohoku University.

References

[1] Reid, H. F., 1911, The elastic-rebound theory of earthquakes, *Univ. Calif. Dept. Geol. Bull.* **6**, 413-444.

[2] Turcotte, D. L., 1992, *Fractals and Chaos in Geology and Geophysics*, Cambridge University Press, Cambridge, 221 pp.

[3] Bak, P., and C. Tang, 1989, Earthquakes as a self-organised critical phenomenon, *J. Geophys. Res.* **94**, 15,635-15,637.

[4] Main, I., 1996, Statistical physics, seismogenesis, and seismic hazard, *Rev. Geophys.* **34**, 433-462.

[5] Vere-Jones, D., 1978, Earthquake prediction: a statistician's view, *J. Phys. Earth* **26**, 129-146.

[6] Vere-Jones, D., and Y. L. Deng, 1988, A point process analysis of historical earthquakes from North China, *Earthquake Res. China* **2**, 165-181.

[7] Zheng, X., and D. Vere-Jones, 1994, Further applications of the stochastic stress release model to historical earthquake data, *Tectonophysics* **229**, 101-121.

[8] Lu, C., D. Harte, and M. Bebbington, 1998, A linked stress release model for historical earthquakes from Japan, submitted to *Earth, Planets and Space.*

[9] David, H., 1998, Documentation for the statistical seismology library, Research Report **98-10**, 1-101 (School of Mathematical and Computing Sciences, Victoria University of Wellington).

[10] Mogi, K., 1985, *Earthquake Prediction*, Academic Press, Tokyo, 355 pp.

[11] Burridge, R., and L. Knopoff, 1967, Model and theoretical seismicity, *Bull. Seismol. Soc. Amer.* **57**, 341-371.

[12] Lu, C., H. Takayasu, A. Yu Tretyakov, M. Takayasu, and S. Yumoto, 1998, Self-organised criticality in a block lattice model of the brittle crust, *Phys. Lett. A* **242**, 349-354.

[13] Kagan, Y. Y., 1994, Observational evidence for earthquakes as a nonlinear dynamic process, *Physica D* **77**, 160-192.

[14] Ben-Zion, Y., 1996, Stress, slip, and earthquakes in models of complex single-fault systems incorporating brittle and creep deformations, *J. Geophys. Res.* **101**, 5677-5706.