# Feasibility study of GeoFEM for solving 100 million DOF problems

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## Abstract

**GeoFEM is a parallel FEM code to analyze solid earth. This report describes large-scaled structural analysis features of the code. One of the purposes for GeoFEM/Tiger version consists in 100 million degree of freedoms (DOF) analysis by parallelized FEM. Here, we discuss the problem to be solved by GeoFEM, the method to enable such a large-scaled structural analysis and its visibility. GeoFEM adopts domain decomposition method (DDM), assuming distributed memory type parallel computers and it solves all degrees of freedom by iterative method such as conjugate gradient. At this stage, the largest linear elastic problem solved by "GeoFEM" is more than 100M (100,000,000) degree of freedoms with reasonable scalability on 1,000 PEs Hitachi SR2201 at University of Tokyo.**

## Introduction

GeoFEM[1],[2] is a parallel FEM system to analyze solid earth. The structure analysis subsystem of GeoFEM deals with solidus behavior of the earth. The solid earth analyses to be solved by GeoFEM system are lithospheric plate motions and earthquake wave propagation problems. The former is a static problem, and it must be considered strong nonlinearity because of geographical complexity. The latter is a dynamic problem including massive time series calculation. We have to handle large scale and complex nonlinearity to solve such a kind of problems. However for the first order approximation, we can analyze those problems by linear system. Although the latter dynamic analysis is usually adopted explicit method, we can apply static techniques excluding time integration scheme. Therefore we discuss here only for large scaled linear analysis, and exclude complexity or dynamic problem.

## Large scale plate motion analysis

According to the plate tectonics theory, the earth is covered with lithospheric plates, which move over the low viscous asthenosphere layer. And main earthly behaviors like earthquake arise from plate boundaries. Especially around the Japanese archipelago, the Eurasia plate, the Pacific Sea plate and the Philippine Sea plate are

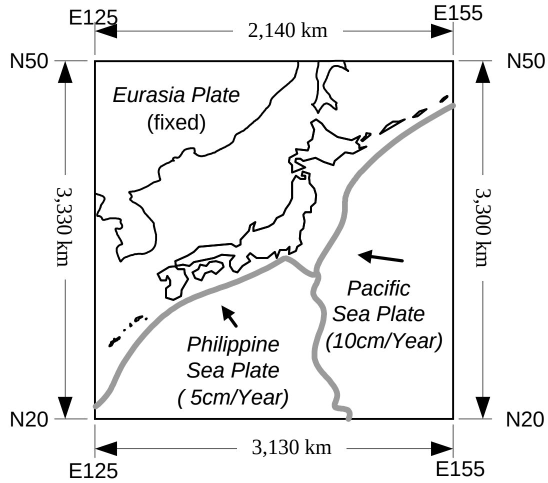


Figure 1: Plate Motions around the Japanese Archipelago. When modeling $3,000 \times 3,000 \times 100$km range crust, we need about 1,000,000 nodes (3,000,000 degree of freedoms) using 10km sided, and about 34,000,000 nodes (100,000,000 DOF) using 3km sided cubic mesh.

complexity intersected. This is the reason why many earthquakes tend to occur in Japan. The movements of the plates become clear by measuring systems such as GPS, which is developed recently. Therefore, when we analyze plate motion around the Japanese archipelago boundary conditioned by measured data and it is possible to study time dependent stress or strain distribution and accumulation at internal or boundary of the plates. These plates motion analyses are expected to predict for the earthquake. However these types of analyses must treat large scale problems because of three dimensional geographical complexities resulting from faults. As Fig.1 we have to deal with large scale problems to analyze geophysical challenges. In terms of this point, we have to use parallelized FEM.

## Analysis methods for parallel FEM

The domain decomposition method (DDM) is generally used for parallel FEM. In this method, we decompose the whole model to some subdomains to assign each PE. At GeoFEM system, the number of decomposed subdomains and PE count remain the same to keep large granularity. When solving degree of freedom (DOF), two types of DDM algorithm are taken. The one is to solve all DOF and the other is to solve boundary DOF. When we adopt the former, iterative solver such as C.G. method is usually used. The latter is still separated to two methods, those are to compose reduced matrix explicitly (substructure method, SSM) or to be converged boundary force by iterative solver[3]. There are several kinds of DDM algorithm and each one has its merits and demerits. As far as large scale analyses are concerned, SSM is not suitable[4]. GeoFEM adopts DDM solving all degrees of freedom by iterative method[5] because of stability and applicability for nonlinear problems. In this method, proper partitioning and pre-conditioning are key technologies for stable and fast calculation. And one of the merits of this method is that we can easily separate solver from other analysis subsystems.

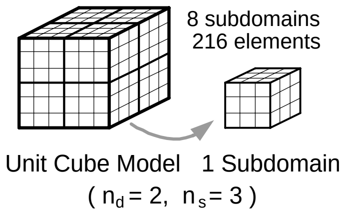


Figure 2: Cubic Model

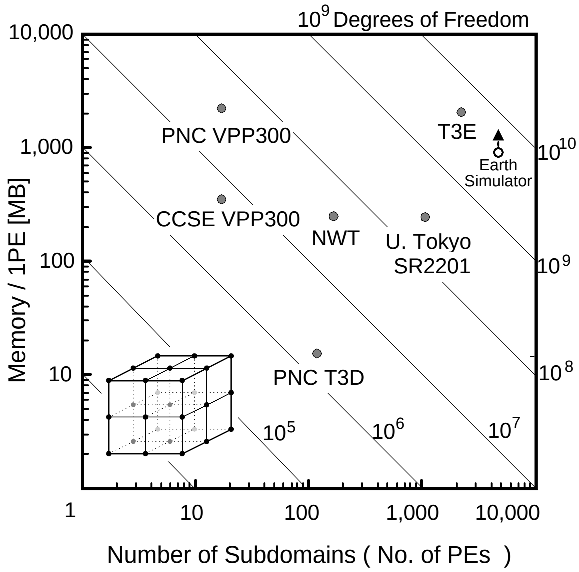


Figure 3: Required Memory Estimation

## Resource estimation for large scaled analysis

In this section, we estimate computer resources for 100 million DOF when using the method described in the previous section. For FEM mesh, we use a simple unit cubic model as Fig.2. The model has unit length sides and it can easily change the DOF to vary division counts of each side. The memory capacity and the elapsed time problems are dominant to deal with large scale FEM analyses and most of all quantity is spent at solver. Therefore we estimate memory capacity and elapsed time required at solver.

**Memory estimation**

At the structural analysis, memory estimation to use iterative solver is predicted by the total amount of memorizing whole stiffness matrix. The term count of the matrix is calculated by the count of connected node at each DOF. For example, term count at a DOF expressed by (DOF per 1 node)×(connected node count). When unknowns setup to displacement, then (DOF per 1 node) becomes 3, for the model in Fig.2; (connected node count per 1 node) is 27. And each term is required a double precision real and one integer type variable, it becomes 12 bytes per term. Therefore, the memory capacity to express whole stiffness matrix becomes 972×(DOF) [byte]. On taking other variables, the rough estimation is expressed below.

$$(\text{Required Memory at Solver}) = 1,000 \times (\text{DOF}) \quad [\text{Byte}] \tag{1}$$

This relationship is shown in the Fig.3. In this figure, the total capacity of the memory becomes (PE count)×(memory per 1PE) considering DDM. When using SR2201 at Tokyo university which has 1,024 PE and 256MB memory per PE, we can calculate 100M DOF analysis.

**Elapsed time estimation**
For elapsed time estimation, we measure wall clock for various DOF by single PE at SR2201. According to the parametric study, the iteration count expressed by power of DOF and the elapsed time per iteration is relative to DOF. These single PE results is expressed by the following equation.

$$\text{(Required Elapsed Time at Solver by 1PE)} = 7.11 \times 10^{-5} \times (\text{DOF})^{1.323} \quad \text{[sec.]} \quad (2)$$

The line of prediction(1PE) on Fig.4 is plotted upper equation. The figure includes 1,000 PE prediction line which is simply divided into one PE value by PE count assuming 100% parallel efficiency. As the result, when SR2201 with 1,000PE is used, we can analyze 100M DOF problem within reasonable time.

## Parametric analyses for cubic model

Fig.4 shows parametric analyses result of parallel computation also. At this stage, the largest linear elastic problem solved by GeoFEM is more than 100M ($33^3 \times 3 \times 1,000 = 107,811,000$.) DOF on 1,000 PEs Hitachi SR2201 at University of Tokyo, by elapsed time 1.5 hour.

Fig.5 shows scalability of the cubic analyses on SR2201 up to 100M DOF. Those analyses fixed $n_s$ and varying PE counts 1, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, $8^3 = 512$, $10^3 = 1,000$. For 1,000 PE analysis, number of DOF becomes about 100M. Because of the Localized preconditioning, parallel efficiency limited about 60%. GeoFEM can show almost linear scalability.

## Conclusion

In this report we focus on large scale structural analysis feature of GeoFEM system, then describe problems to be analyze, method of parallelization and computer resource estimation. At the conclusion we can show 100M DOF analysis is possible by the present parallel computer, SR2201 with 1,024 PE. The Earth simulator which is going to be completed at AD 2001, aims at 30Tflops peak speed. This speed is 100 times faster than that of SR2201. By the present parallel computer, 100M DOF analysis is limited to simple linear static analysis, however when the Earth simulator is completed, we will analyze large scale and complex geophysical phenomena by GeoFEM.

## Acknowledgement

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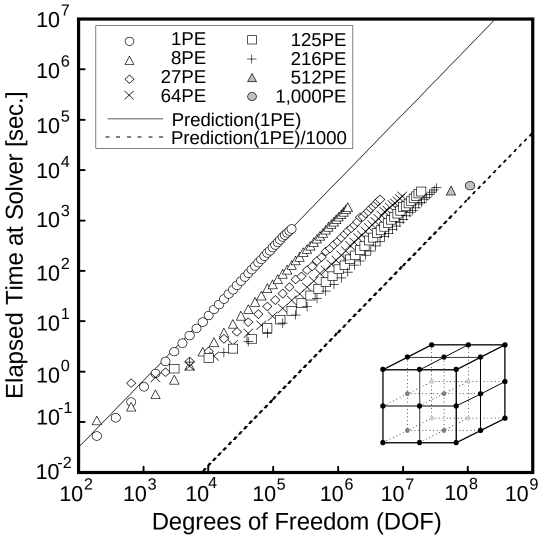


Figure 4: Elapsed Time at Solver.

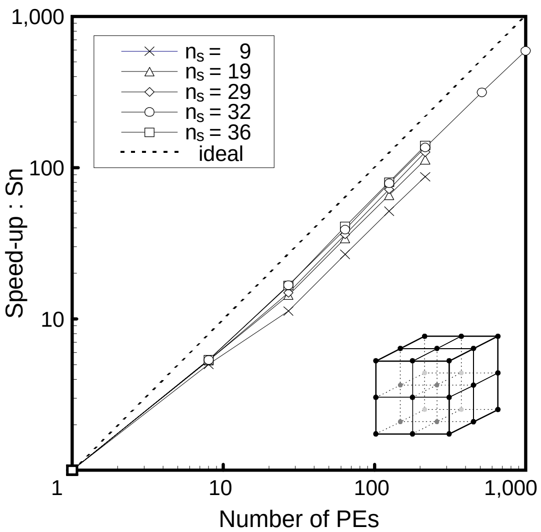


Figure 5: Scalability.

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