

FEM simulation of dynamic sliding of faults connecting static deformation process to dynamic process

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Introduction

The objective of present study is to develop a finite element method (FEM) software system toward a large-scale computation of following seismic processes:

- 1) Accumulation of the stress around active faults in the crust, which is induced by a subduction of plates in a long time span (static deformation process);
- 2) Onset of an earthquake due to a dynamic sliding on the faults, which is induced by the tangential stress component acting on the fault surface, together with propagation of the seismic wave in a complex media (dynamic deformation process).

In order to simulate above-mentioned seismic processes, we have started to develop two different numerical codes: a static FEM code [1] and a dynamic FEM code [2]. For the both FEM codes, three nonlinearities are involved. They are *material nonlinearity*, *geometrical nonlinearity* and *contact nonlinearity*.

These nonlinearities have to be taken into account especially in the simulation of a long term deformation process of solid earth, since the material behavior of seismic media at a finite strain is described by nonlinear equation like elasto-visco-plasticity, and also friction behavior on the plate interface and on the active fault surface must be expressed by a complex nonlinear equation, such as functions of sliding distance, sliding rate, temperature and depth.

Simulation sequence of modeling processes is illustrated in Fig.1. In the static analysis, the deformation of complex media caused by the motion of plates is simulated in a wide range of space, in which the plate interfaces and active faults are contained. During this process the shear stress component acting on the surface of all faults is detected, and when the accumulated shear stress reaches the strength of fault (critical value in the friction constitutive model of fault) at any point on the fault surface, the stress and strain data of entire domain calculated by the static FEM are transferred to the dynamic FEM code. In the dynamic analysis, the onset of earthquake and the propagation of seismic wave are simulated. Until now, we have already developed a parallel dynamic FEM code to simulate earthquake phenomena including the dynamic sliding of faults and the seismic wave propagation. A slip-dependent friction law has been introduced into our FEM code, and the viscous boundary condition [3] has been employed as an absorbing condition to suppress the

reflection of seismic waves against the artificial boundaries, which is introduced in order to limit the analytical domain. As the parallel computational algorithm, the domain decomposition method is applied on both static and dynamic FEM simulation.

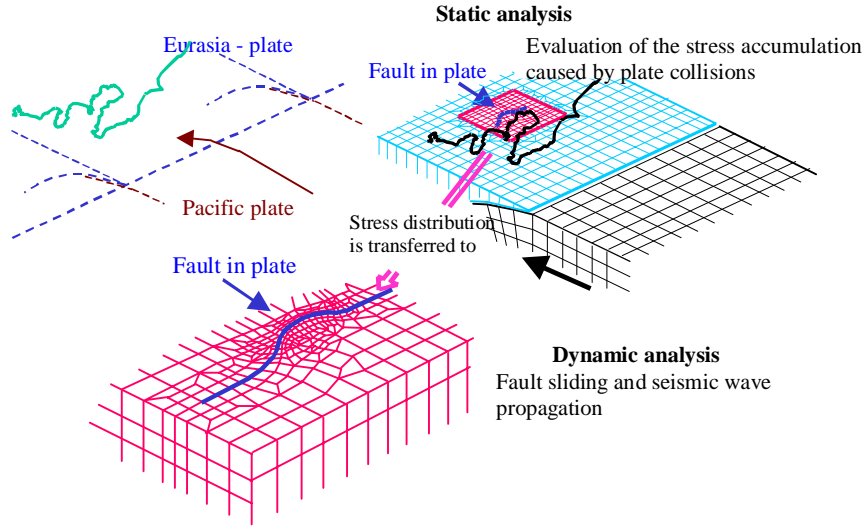


Figure 1. Modeling the process of static and dynamic analyses for earthquake initiated ground motion

FEM formulation of multi-body contact problem

The Cauchy equation of motion is described as follows. The dynamic equilibrium equation on the crust is as following;

$$\nabla S = \rho \text{Siin}\Omega, \quad (1)$$

where ρ is density, u is the displacement vector, \ddot{u} is the acceleration vector, S is the stress tensor. If δu is assumed as a virtual displacement vector, then the principle of virtual work for multi-bodies at current configuration is obtained.

$$\sum_{L=1}^N \left\{ \int_V \rho \ddot{u} \delta u dV + \int_V \delta e dV \right\} = \sum_{L=1}^N \left\{ \int_{S_f} \delta u f^s dS \right\} + \sum_{L=1}^N \int_{S_c} \delta u^c f^c dS \quad (2)$$

$L=1, \dots, N$ means that N bodies are in contact at current configuration. The part given in the braces corresponds to usual terms and the last summation sign gives the contribution of the contact forces. If the discretization is made by introducing an isoparametric shape function, then Eq. (2) can be written with a matrix form in following form, here a artificial matrix C is introduced to describe the material damping.

$$\{M\ddot{u} + C\dot{u} + Ku\} = \{F^s\} + F^c, \quad (3)$$

where F^s and F^c express surface force and the contact force due to friction caused by the faults on each nodes, respectively. Since the period of stress accumulation process is of a

long time span, this process is generally assumed as a quasi-static process, therefore, the effect of acceleration and velocity can be ignored in static analysis.

$$\{Ku\} = \{F^s\} + F^c, \quad (4)$$

On the other hand, the dynamic explicit FEM formulation is obtained by using the central difference method for time integration of Eq. (3).

$$\left\{ \left(\frac{M}{\Delta t^2} + \frac{C}{2\Delta t} \right) u_{t+\Delta t} \right\} = \left\{ F^{ext} - F^{int} + \frac{2M}{\Delta t^2} u_t - \left(\frac{M}{\Delta t^2} - \frac{C}{2\Delta t} \right) u_{t-\Delta t} \right\} + F^c, \quad (5)$$

Only the surface force is included in the external force, that is $F^{ext} = F^s$, since we assume that the areas S_p , on which force boundary conditions are prescribed, are not part of the contact surface S_c . In dynamic FE code, the following relation is used instead of using

$$Ku = F^{int}, \quad F^{int} = \int_V B^T \sigma dV, \quad (6)$$

Now the problem is how to calculate the contact force F^c correctly.

Contact force calculation algorithm

The contact force F^c can be decomposed into two components referred to normal direction n and sliding direction s on S_c .

$$F^c = F_n^c n + F_t^c s = (F^c n)n + (F^c s)s, \quad (7)$$

In order to calculate F^c , the following relations satisfied on contact surface are used, referred to Fig.2.

Geometrical condition

$$x^{(1)} = x^{(2)} \quad \text{and} \quad n^{(1)} = -n^{(2)}, \quad (8)$$

Kinematics condition

$$u^{(1)} \cdot n^{(1)} = u^{(2)} \cdot n^{(2)} \quad \text{and} \quad \dot{u}^{(1)} \cdot n^{(1)} = \dot{u}^{(2)} \cdot n^{(2)}, \quad (9)$$

Kinetic condition

$$F^{c(1)} + F^{c(2)} = 0, \quad (10)$$

$$F^{c(1)} \cdot n^{(1)} \leq 0 \quad \text{and} \quad F^{c(2)} \cdot n^{(2)} \leq 0 \quad (11)$$

$$F^c \cdot s = \varphi(w, x) \quad (12)$$

where $\varphi(w, x)$ is friction force that depends on a friction constitutive model, and w is the relative displacement.

In the finite element analysis, the contact condition for normal direction n is described by the displacement condition, Eq. (8) and Eq. (9), from which the contact force is calculated. The condition of tangential direction is treated as the traction condition, Eq.(12), from which the sliding is found. In finite element fault analysis, slip-dependent friction law or rate- and state-dependent friction law are widely employed as friction constitutive model.

Penalty method for normal contact force calculation

For an explicit dynamic FE code, it is very convenient to use a so-called Penalty method to calculate the normal contact force.

$$F_n^c = -p_n g, \quad (13)$$

where g is penetration of the master node to the slave contact surfaces, p_n is a constant coefficient called penalty factor and defined as follows.

$$p_n = \frac{K A^2}{V}, \quad (14)$$

K is volume modulus of the fault material, A is the element area and V is the element volume.

Slip-dependent friction law for tangential contact force calculation

In the present study, a slip-dependent friction law [4] is used to compute the tangential contact force.

$$\tau(w, x) = \tau_b(x) \frac{w}{w_c(x)} \exp\left[1 - \frac{w}{w_c(x)}\right], \quad (15)$$

where, $\tau_b(x)$ and $1/w_c(x)$ indicate the peak shear stress and slip-weakening rate at point \mathbf{x} , respectively, which are most important parameters on slip-dependent friction law. The relative displacement w increases very slowly like quasi-static process until it reaches the critical value $w_c(x)$. Once the tangential component of the surface stress reaches the $\tau_b(x)$ the faults slid dynamically. At this process the energy released is the shaded portion on Fig.3 [5,6,7,8,9]. Therefore, the friction model Eq. (15) can be simplified as follow when the seismic earthquake is calculated numerically

$$\tau(w, x) = \tau_b(x) \left(\frac{5w}{D_c} + 1\right) \exp\left[-\frac{5w}{D_c}\right], \quad (16)$$

Here,

$$\tau_b(x) = \mu_s(x) \sigma_n. \quad (17)$$

In order to implement Eq. (16) into a FE code, we rewrite Eq. (16) with the form of Coulomb's friction law as follow [10].

$$\tau(w, x) = \mu_d(w, x) \sigma_n, \quad (18)$$

$$\mu_d(w, x) = \mu_s(x) \left(\frac{5w}{D_c} + 1\right) \exp\left[-\frac{5w}{D_c}\right]. \quad (19)$$

If we assume the tangential contact force has the same relation with shear stress, then

$$F_t^c(w, x) = \mu_d(w, x) F_n^c. \quad (20)$$

Finally the displacement for each time step can be solved by substituting the contact condition, Eq. (13) and (20), into Eq. (5).

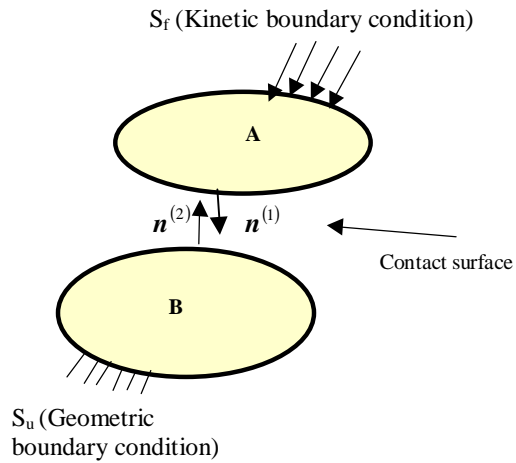


Figure 2. Contact between two bodies

Dynamic simulation of fault sliding and propagation of seismic wave

The dynamic motion of fault depends on the friction laws if the active fault has already existed.

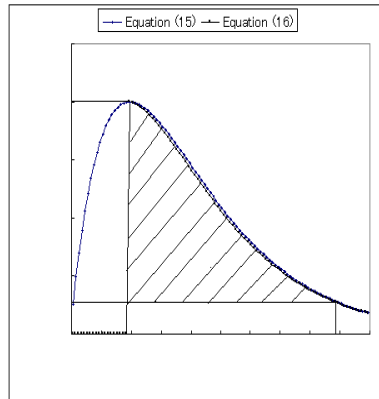


Figure 3. Slip-dependent friction law of fault

In this case, the tectonic stress accumulated in a long time quasi-static process is not relaxed entirely during the earthquake due to the dynamic friction of the fault. Since this residual stress affects the earthquake cycle significantly, the analysis of dynamic process is very important not only for preceding the onset of the earthquake and propagation of seismic wave, but also for estimating the effects of dynamic sliding process to earthquake cycle. Therefore a number of analysis with FEM have been done recently [11,12]. However,

the seismic wave propagation simulation coupled with fault sliding analysis is less found yet.

The FEM calculation is carried out on IBM-SP2 distribute memory parallel computer of Japan Atomic Energy Research Institute (JAERI). In order to simulate the propagation of seismic wave during 7.2 seconds, $7.2/0.0024=300$ steps, 13 min. of calculation time and 25 Mbytes of memory were necessary with 16 processes.

The material parameters used in this study are expressed in Table 1, and FEM parameters for fault analysis and wave propagation simulation [13] are as follows.

Material	GRANITE
S-wave velocity V_s (m/s)	3550
P-wave velocity V_p (m/s)	6230
Density (Kg/m ³)	2619
Young modulus (Pa)	8.3148×10^{10}
Poisson ratio	0.26

(i) For fault analysis
 Analysis domain : $100 \times 100 \times 100$ (mm³)
 Mesh division : $10 \times 10 \times 10 = 1000$
 Time increment : 1.82×10^{-7} (s)
 (ii) For wave propagation simulation
 Analysis domain : $28 \times 25 \times 12.5$ (Km³)
 Mesh division : $35 \times 50 \times 25 = 43750$
 Time increment : 0.0024 (s)

Table 1. Parameters of FEM model

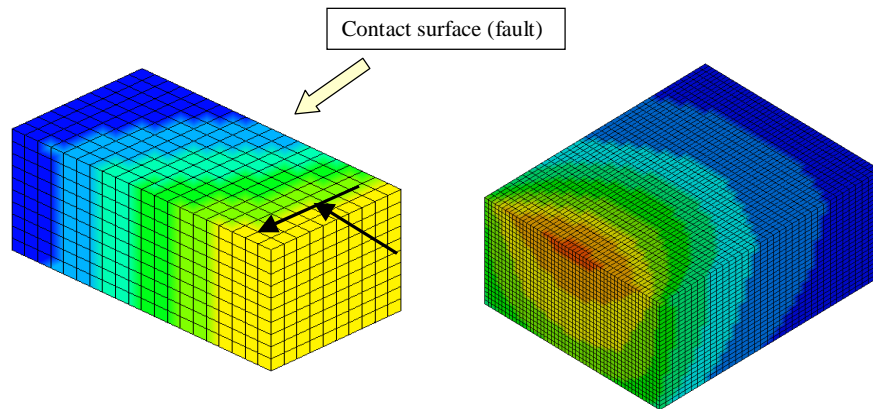


Figure 4. Simulation results of fault sliding and propagation of seismic wave

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