A unified comprehension for fracture of intact rock, frictional slip failure, and earthquake rupture, and scaling of scale-dependent physical quantities inherent in the rupture

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There are increasing amounts of evidence that earthquake rupture is a mixed process between what is called frictional slip failure and fracture of intact rock mass. In fact, there is a strong possibility that what is called a 'barrier' or 'asperity' on earthquake faults is a local patch of high rupture growth resistance whose strength equals the strength of intact rock. In addition, the lithostatic pressure and temperature are high enough at crustal depths corresponding to the brittle-plastic transition regime, so that the shear frictional strength conforms to the shear fracture strength of intact rock at these depths. Shear fracture of intact rock is the upper end member of frictional slip failure on a preexisting fault. If, therefore, there is a constitutive law that governs the earthquake rupture, the constitutive law should be formulated as a unifying constitutive law that governs both frictional slip failure and shear fracture of intact rock mass.

There are two types of physical quantities inherent in shear rupture: scale-dependent quantities, and scale-independent quantities. The scale-dependent quantities include the shear fracture energy G_c, the breakdown zone size X_c and its duration T_c, the nucleation zone size L_c and its duration t_c, the slip acceleration $\ddot{D}$, and the cutoff frequency $f^{\mathrm{s}}_{\mathrm{max}}$ of the power spectral density of the slip acceleration versus time record observed at a position on the fault. Specifically, for instance, $G_c = 10^6 - 10^8 J/m^2$ has been evaluated for major earthquakes, while $G_c = 10^4 - 10^5 J/m^2$ for shear fracture of intact rock sample under lithospheric conditions, and $G_c = 0.1 - 1 J/m^2$ for stick-slip dynamic failure on preexisting faults. Both L_c and t_c for large-scale earthquake rupture also differ greatly from those for shear rupture of small scale in the laboratory, as corroborated by existing data. How can these scale-dependent physical quantities be treated unifyingly in quantitative terms ? To answer this question, the introduction of the constitutive law is indispensable; however, the constitutive law is needed to be formulated appropriately so as to scale the above scale-dependent physical quantities consistently, and thereby a unified comprehension should be provided for the shear rupture of any size scale - small scale in the laboratory to large scale in the Earth as an earthquake source.

The shear rupture include a phase of stable, quasi-static rupture growth and a phase of unstable, dynamic high-speed rupture. These two phases should also be treated quantitatively by a single constitutive law, because they are part of the same rupture process.

All the above physical requirements lead to the conclusion that the constitutive law should be formulated primarily as a slip-dependent law. Recent experiments on fracture of intact rock and frictional slip failure have led to a new finding that the slip-dependent constitutive law parameters are mutually not independent, but related to one another by the following universal relation:

$$\frac{\Delta\tau_b}{\tau_p} = \beta\left(\frac{D_c}{\lambda_c}\right)^M \quad ,$$

where τ_p is the peak shear strength, $\Delta\tau_b$ is the breakdown stress drop, D_c is the critical slip displacement, λ_c is the characteristic length representing geometric irregularity of the rupturing surfaces, and β and M are non-dimensional constants. The above relation plays a fundamental role in unifying shear fracture of intact rock, and frictional slip failure, and as a consequence, earthquake rupture.

It has been shown theoretically in the framework of fracture mechanics based on a slip-dependent constitutive law that physical quantities inherent in the shear rupture are expressed in terms of the constitutive law parameters, and that the scale-dependent physical quantities are directly related to D_c, which is a scale-dependent constitutive parameter. For instance, G_c is expressed explicitly in terms of the slip-dependent constitutive law parameters, $\Delta\tau_b$ and D_c, as (Palmer and Rice, 1973[4]; Ohnaka and Yamashita, 1989[3]):

$$G_c = \int_0^{D_c} [\tau(D) - \tau_r] dD = \frac{\Gamma}{2}\Delta\tau_b D_c \quad ,$$

where $\tau(D)$ is a slip-dependent constitutive relation, τ_r is a residual friction stress level, and Γ is a numerical parameter dependent on a specific form of the slip-dependent constitutive relation. The breakdown zone size X_c and the critical size L_c of the nucleation zone are also expressed in terms of the constitutive law parameters, $\Delta\tau_b$ and D_c, as (Ohnaka, 1996[2]):

$$X_c = L_c = \frac{1}{k}\left(\frac{\mu}{\Delta\tau_b}\right) D_c \quad ,$$

where k is a non-dimensional parameter depending on the rupture velocity V, and μ is the rigidity. This relation is rewritten in terms of λ_c as:

$$X_c = L_c = \frac{1}{k}\left(\frac{1}{\beta}\right)^{\frac{1}{M}} \frac{\mu}{\Delta\tau_b}\left(\frac{\Delta\tau_b}{\tau_p}\right)^{\frac{1}{M}} \lambda_c \quad .$$

Similarly, the slip acceleration is written as (Ida, 1973[1]; Ohnaka and Yamashita, 1989[3]):

$$\ddot{D} = \frac{\Gamma^2\phi''}{\pi^4}\left(\frac{V}{C(V)}\frac{\Delta\tau_b}{\mu}\right)^2 \frac{1}{D_c} \quad ,$$

where ϕ'' is the non-dimensional slip acceleration and $C(V)$ is a known function of V, and the cutoff frequency $f^{s}_{\max}$ at the source is expressed as (Ohnaka and Yamashita, 1989[3]):

$$f^{s}_{\max} = \frac{h}{\pi^2}\left(\frac{V}{C(V)}\frac{\Delta\tau_b}{\mu}\right)\frac{1}{D_c} \quad ,$$

where h is a numerical constant (~ 1.876). In addition, the seismic moment M_o of a mainshock earthquake can be expressed in terms of the slip-dependent constitutive law parameters, $\Delta\tau_b$ and D_c, as:

$$M_o = c_1 c_2 \left(\frac{\kappa\Gamma}{2}\right)^3 \left(\frac{S_{\mathrm{A1}}}{S}\right)^3 \left(\frac{\mu}{\Delta\tau}\right)^3 \left(\frac{\Delta\tau_b}{\Delta\tau}\right)^3 \mu D_c^3 \quad ,$$

where c_1, c_2, and κ are constants, S is the fault area, S_{A1} is the area of the geometrically largest patch size of high resistance against rupture growth on the fault S, and τ is the stress drop averaged over the fault area S.

These theoretical relations show that the scale-dependent physical quantities are consistently scaled in terms of one of the constitutive law parameters, D_c.We thus conclude that scale-dependency of scale-dependent physical quantities is commonly ascribed to scale-dependency of D_c, and therefore, the fundamental question for the scale-dependency is why D_c is scale-dependent, because the other constitutive parameters, τ_p and $\Delta\tau_b$, are scale-independent. The answer for this question is simple. An earthquake source is shear rupture on a preexisting fault in the seismogenic layer; however, such a preexisting fault itself inherently exhibits geometric irregularities and mechanical inhomogeneities of various scales in the fault zone. Consider for instance a case where a local patch of high rupture growth resistance (which may be called asperity) in the fault zone is broken down. D_c is by definition the slip displacement required for the breakdown of the local patch in this case, and a large amount of slip displacement is needed for the breakdown of the patch of geometrically large size, while only a small amount of slip is necessary for the breakdown of the small patch size. A characteristic scale governing the breakdown process during earthquake rupture is practically determined by the geometric size of a representative patch of high rupture growth resistance on the fault. A local patch of high rupture resistance may be attained at portions of fault bend or stepover, at interlocking asperities of large size in the fault zone, and/or at portions of adhesion (or cohesion) healed between the mating fault surfaces during the interseismic period. The present discussion can be paraphrased in terms of the characteristic length λ_c in the slip direction representing the geometric size of a largest patch of high rupture growth resistance in the fault zone, since D_c scales with λ_c.

The scaling relations derived above are validated by laboratory data on fracture of intact rock and frictional slip failure (stick-slip), and field data on earthquakes. It is thus concluded that a unified comprehension can be provided for shear rupture of any size scale if the constitutive law for shear rupture is formulated as a slip-dependent law.

References

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