Long-range stress re-distribution resulting from damage in heterogeneous media

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Abstract

It has been shown in cellular automata simulations and data analysis of earthquakes that declustered or characteristic large earthquakes may occur with a long range stress re-distribution but aftershocks or power law scaling events do not. In order to understand the long-range stress re-distribution, we proposed a linear-elastic but heterogeneous-brittle model. The stress re-distribution in the heterogeneous-brittle medium implies a longer-range interaction than that in elastic medium. Therefore, it is supposed that the longer-range stress re-distribution resulting from damage in heterogeneous media may be a possible mechanism governing mainshocks.

Introduction

Recently, the significance of long-range stress re-distribution in understanding of earthquake mechanism has drawn a great deal of attention [1-5]. Various versions of cellular automata (CA) with long-range stress re-distribution for earthquake faults were widely used in studies. But understanding what the nature of the long-range interaction and its significance is by no means an easy problem. Different research groups have different understandings. For instance, Klein et al [2] stated that "linear elasticity yields long-range stress tensors for a variety of geological applications" and "for a two-dimensional dislocation in a three-dimensional homogeneous elastic medium, the magnitude of the stress tensor goes as ~ $1/r^3$." They noticed that "while geophysicists do not know the actual stress tensors for real faults, they expect that long-range stress tensors, which are similar to the ~ $1/r^3$ interaction, apply to faults." Moreover, they stressed that "it is suspected that microcracks in a fault, as well as other 'defects' such as water, screen the ~ $1/r^3$ interaction, leading to a proposed ~ $e^{-\alpha r}/r^3$ interaction, where α<<1, implying a slow decay to the long-range interaction over the fault's extent [2]."

Contrary to this, Weatherley et al [3] pointed out in their cellular automata that "the interaction exponent (p in ~ $1/r^p$) determines the effective range for strain re-distribution in the model. The effective range decreases rapidly as the exponent (p) increases. The event size-distribution illustrates three different populations of events in the dissipative healing models (two-dimensional models):

- Characteristic large events ($p < 1.5$)
- Power-law scaling events ($1.5 < p \leq 2.0$)
- Overdamped, no large events ($p > 2.0$)".

Physically, the existence of cracks and other "defects" like water may have two opposite effects on stress re-distribution. One is to screen stress and then lead to a shorter-range re-distribution, as claimed by Klein et al [2]. On the other hand, the stress balance requires a compensational increase of the stress beyond the "defects." This implies a longer-range re-distribution of stress.

Recently, Knopoff [5] investigated the magnitude distribution of declustered earthquakes in Southern California. He concluded that the presumed universality of the scale independence of the complete suite of local earthquakes is attributed to the universality of the properties of aftershocks.

All of the results remind us that a declustered or characteristic large earthquake may occur relevant to a longer-range stress re-distribution. But aftershocks do not. Then, instead of the commonly used homogeneous linear elastic theory, could we find some possible alternative models with longer-range stress re-distribution?

Possible long-range stress re-distribution in heterogeneous media

It is well known that the main terms of stresses in a homogeneous linear elastic medium with are ~ $1/r^3$ and ~ $1/r^2$ for a spherical void in three dimensions and a cylindrical hole in two dimensions, respectively. Now, let us examine the stress re-distribution owing to a void in a linear-elastic but heterogeneous-brittle medium to find out the effect of microdamage resulting from the heterogeneity on stress re-distribution.

It is supposed in the model that every mesoscopic element has the same elastic moduli, like Young's modulus E and Poisson ratio ν, but various breaking strengths σ_c or ε_c. Moreover, the strength of the element follows a distribution function,

$$h(\varepsilon_c) = \begin{cases} 0; & when \quad \varepsilon_c < \varepsilon_c * \\ \frac{q-1}{\varepsilon_c *}(\frac{\varepsilon_c}{\varepsilon_c *})^{-q}, q > 1; & when \quad \varepsilon_c \geq \varepsilon_c * \end{cases} . \quad (1)$$

We have to confess that we do not know the actual distribution of strength in geological media. However, the simple distribution looks qualitatively reasonable and provides some simple understanding of the stress re-distribution in heterogeneous media. In fact, the heterogeneity must imply some intrinsic length scales relevant to structures of geological media. But, as first approximation, we use local mean field to deal with the problem. In this way, the intrinsic length scales are eliminated in the approximation and the stress-strain relation in one-dimensional stress state is, as shown in Fig.1,

$$\sigma = \begin{cases} E\varepsilon; & when \quad \varepsilon < \varepsilon_c * \\ E\varepsilon_c *(\frac{\varepsilon}{\varepsilon_c *})^{2-q}; & when \quad \varepsilon \geq \varepsilon_c * \end{cases} \quad (2)$$

$$D = \begin{cases} 0; & when \quad \varepsilon < \varepsilon_c * \\ 1-(\frac{\varepsilon}{\varepsilon_c *})^{1-q}; & when \quad \varepsilon < \varepsilon_c * \end{cases} \tag{3}$$

In accord with the damage mechanics, the effect of damage can be described by the reduced modulus,

$$E' = E(1-D) = E\bar{\varepsilon}^{1-q}, \tag{4}$$

where $\bar{\varepsilon} = (\frac{\varepsilon}{\varepsilon_c *})$.

In particular, it is presumed that the damage or the reduced moduli are governed by maximum strain, i.e. the circumferential strain ε_θ. We call this the θ-model. Then, when $\bar{\varepsilon}_\theta > 1$, the elastic-brittle constitutive relation in cylindrical configuration (2-D plane stress), becomes

$$\bar{\sigma}_r = [\bar{\varepsilon}_r + \nu\bar{\varepsilon}_\theta]\bar{\varepsilon}_\theta^{1-q} \tag{5}$$

$$\bar{\sigma}_\theta = [\nu\bar{\varepsilon}_r + \bar{\varepsilon}_\theta]\bar{\varepsilon}_\theta^{1-q}, \tag{6}$$

where $\bar{\sigma} = (1-\nu^2)\sigma / E$. In the following we will ignore the bar above all dimensionless variables. The balance equation in cylindrical configuration (2-D) is

$$\frac{d\sigma}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{7}$$

Substitution of the strain definition and the elastic-brittle relation (5-6) into the balance equation (7) leads to a non-linear ordinary differential equation of displacement *u*,

$$u'' + (1-q)\frac{u'^2}{u} + (q+\nu-\nu q)\frac{u'}{r} + (\nu q - 1 - \nu)\frac{u}{r^2} = 0, \tag{8}$$

where u' denotes differentiation with respect to radial distance *r*. It is worth noticing that when *q*=1, Eq. (8) reduces to the linear elastic version. But, when *q* is greater than unity, the non-linear second term in Eq. (8) plays a significant role. We use the following non-linear transformation to linearize equation (8):

$$u = v^{\frac{1}{2-q}}. \tag{9}$$

Then there is power type solution

$$v = Ar^{-\beta}, \tag{10}$$

where *A* is an arbitrary constant and β has two values

$$\beta = \begin{cases} q-2 \\ 1+\nu-q\nu \end{cases}. \tag{11}$$

Hence, the stresses, either circumferential or radial, will be in the form

$$\sigma = [O(1) + O(\frac{\varepsilon_r}{\varepsilon_\theta})]\varepsilon_\theta^{2-q} = [O(1) + O(\frac{\varepsilon_r}{\varepsilon_\theta})][A_1 + A_2 r^{-p}], \tag{12}$$

where A_1 and A_2 are two arbitrary constants. One can verify that the term

$$O(\frac{\varepsilon_r}{\varepsilon_\theta}) \approx O(1)\,. \tag{13}$$

and p is

$$p = \beta + 2 - q = \begin{cases} 0 \\ 3 + \nu - (1+\nu)q \end{cases}. \tag{14}$$

The power index p in stress re-distribution approaches 2 as q approaches 1, and p decreases with increasing q. That is to say, stress re-distribution in the heterogeneous elastic-brittle medium has longer interaction range with stronger heterogeneity.

Similarly we obtained the solution for the three-dimensional (3-D) configuration and the corresponding exponent p, see Table 1:

$$p = \beta + 2 - q = \begin{cases} 0 \\ 2\frac{2-\nu}{1-\nu} - \frac{1+\nu}{1-\nu} q \end{cases} \tag{15}$$

Discussion

There are two main assumptions made in the θ - model: that the Poisson ratio ν remains invariant and the circumferential strain ε_θ governs the reduced moduli. In order to check the assumptions of the θ-model, we calculate an alternative model, the mixed model, which consists of elastic and damaged deformations. In the two-dimensional case, it is

$$\overline{\sigma}_r = [\overline{\varepsilon}_r + \nu \overline{\varepsilon}_\theta], \tag{16}$$

$$\overline{\sigma}_\theta = [\nu \overline{\varepsilon}_r + \overline{\varepsilon}_\theta] \overline{\varepsilon}_\theta^{1-q}\,. \tag{17}$$

In addition, finite element simulations in two-dimension were made in accordance with the assumptions of the θ- model. The results are in good agreement, see Fig. 2.

In summary, in order to understand long range stress re-distribution, we proposed a linear-elastic but heterogeneous-brittle model. The stress re-distribution in the heterogeneous-brittle medium implies a longer-range interaction. Therefore, it is supposed that the long-range stress re-distribution resulting from damage in heterogeneous media with intrinsic length scales can quite possibly be a mechanism governing mainshocks.

Acknowledgments

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Table 1. The formula and values of power exponent p in approximate power law, $\sigma \approx r^{-p}$ in three and two dimensional configurations.

Q	P (3 – D)	P (2 – D)
	$p = 2\frac{2-\nu}{1-\nu} - \frac{1+\nu}{1-\nu}q$	$p = 3+\nu-(1+\nu)q$
When ν = 1/4		
1	3	2
1.2	2.66	1.75
1.5	2.16	1.37
1.75	1.75	1.06

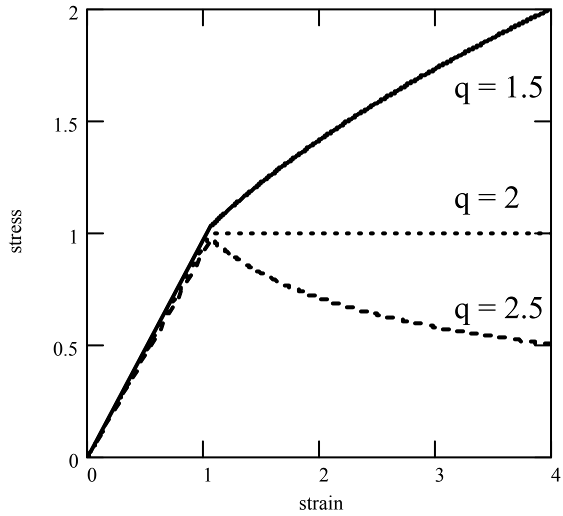

Fig.1 The one-dimensional stress and strain relation of the linear-elastic but heterogeneous-brittle model with different q values. It shows that the greater the mesoscopic strength scatter is, the softer the model becomes.

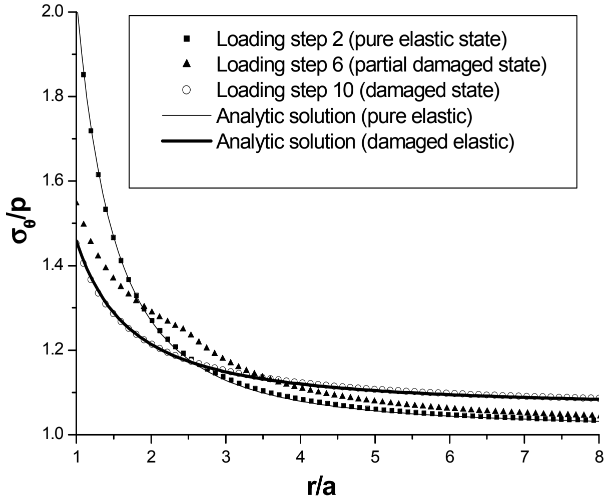

Fig. 2 The variations of circumferential stress with radial distance for the elastic-heterogeneous brittle medium with parameters q=1.5 and ν=0.25 for various loading steps. The points are the FE results. The two solid lines are the analytical results of elastic and the damaged elastic-brittle models respectively. The agreements between FE and analytic solutions are very good. The dotted line (▲) in between is the FE result for the state of partly elastic(outer part) and partly damaged(inner part).