Does The Scaling of Strain Energy Release With Event Size Control The Temporal Evolution of Seismicity?

Steven C. Jaumé[(1)]

(1) Department of Geology and Environmental Geosciences, College of Charleston, Charleston, South Carolina, USA, (e-mail: jaumes@cofc.edu, phone: +1-843-953-1802, fax: +1-843-953-5446)

Abstract

What parameters control the evolution of seismicity in a distributed fault system? Results from statistical models of regional seismicity suggest that the scaling of strain energy release in the crust with earthquake magnitude may be an important factor. Models where strain release scales according to Benioff strain (strain energy $\propto 0.75M$) show effectively linear strain release through time and no evolution in event statistics, similar to cellular automaton (CA) models with self-organized critical (SOC) behavior. Conversely, when strain release is scaled according to seismic moment (strain energy $\propto 1.5M$) the rate of strain release accelerates up to the time of occurrence of large magnitude events and drops thereafter, similar to CA models that show intermittant critical behavior. This raises the question of how strain energy release scales with magnitude in both models and the real earth. Preliminary results of internal strain energy changes associated with earthquakes on a strike-slip fault imbedded in an elastic halfspace suggest that strain energy release $\propto 2.2M$; i.e., that the largest earthquakes do most of the work in removing stored elastic energy from the crust. If this result is a general one then it supports an intermittant critical model of regional seismicity over a SOC model.

Introduction

What controls the space-time-size evolution of seismicity in a deforming region? Do the largest earthquake events occur randomly or is there some pattern (quasi-periodic or otherwise) to their behavior? These questions, among others, have been the focus of considerable observational (e.g., Jaumé, 2000) [6], theoretical (e.g., Rundle *et al.*, 1999) [9] and simulation research (e.g., Weatherley *et al.*, 2000) [12].

Here I briefly compare results from two different types of simulation models, a cellular automaton model based upon simplified physics of a fault system (Weatherley *et al.*, 2000) [12] and a statistical model of the temporal evolution of regional seismicity (Jaumé and Bebbington, 2000) [5]. Event and energy histories in these models suggest the evolution of seismicity is in part controlled by the scaling of the system energy release with event size. I then construct a simple elastic dislocation model of a strike-slip fault and investigate how strain energy release in the model scales with increasing moment magnitude.

Comparison of Stress Release and Cellular Automaton Results

Details of the stress release (SR) (Vere-Jones *et al.*, 2001) [11] and cellular automaton (CA) (Weatherley *et al.*, 2000) [12] models can be found elsewhere. For the purposes of this discussion I wish to show that both models can reproduce what appear to be two distinct modes of behavior; one where the system energy remains nearly constant and the largest events occur randomly, called continous self-organized criticality (SOC) and the other where the largest events significantly decrease the energy of the system and the largest events occur quasi-periodically, called critical point behavior (CP) (Figure 1). In the SR models either the event occurrence rate or the upper magnitude limit (or both) can be a function of the system energy level. However, it was found that the scaling of the energy drop with magnitude exerted the greatest influence over the evolution of seismicity in the models; stress drops scaled according to $S_i = 10^{2.4+0.75M_i}$ (Benioff strain) produced SOC behavior; those scaled according to $S_i = 10^{9.0+1.5M_i}$ (seismic moment) produced CP behavior.

The results described above motivate the following question: How does an earthquake change the energy state of the fault system in which it is imbedded? Earthquakes are a result of the release of potential energy stored in elastically strained crust (i.e., strain energy). Thus the change in internal strain energy resulting from an earthquake rupture is the natural scalar quantity to use in SR-type models. However, a straightforward scaling relationship between strain energy release and earthquake magnitude does not exist.

Strain Energy Release Model

The amount of strain energy released by an earthquake cannot be directly estimated from seismograms since the radiated seismic energy is dependent only upon the stress drop and not upon the initial stress (Aki and Richards, 1980) [1]. This means that two earthquakes with the same fault displacement can have very different strain energy releases.

My interest here is not only the absolute level of strain energy release but more importantly how it scales as earthquake magnitude increases. Therefore, I have chosen to model the strain energy change by assuming a fault surface pre-loaded by tectonic displacements and calculating the strain energy stored in the deformed volume before and after an earthquake displacement function is added to the model.

The strain energy in an elastically deformed volume is (Jaeger and Cook, 1979) [4]:

$$W = \frac{1}{2} \int_V \sigma_{ij} \epsilon_{ij} dV \tag{1}$$

where σ_{ij} is the stress tensor, ϵ_{ij} is the strain tensor and V is the strained volume. If this integral can be evaluated before (W_{int}) and after (W_{fin}) an earthquake event, the total change in strain energy ΔE_{st} due to the earthquake can be found:

$$\Delta E_{st} = W_{int} - W_{fin} \tag{2}$$

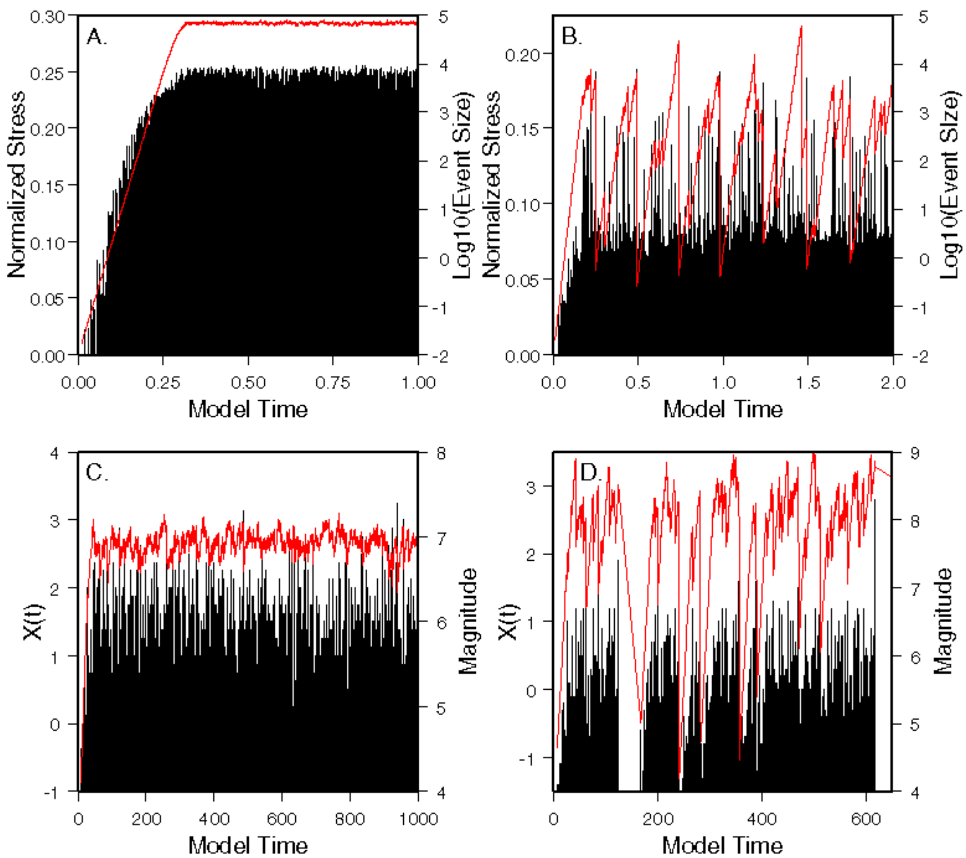

Figure 1: Comparison of energy and event histories in cellular automaton (CA) and stress-release (SR) models. A) CA model that shows self-organized critical behavior, B) CA model that shows intermittent critical behavior, C) SR model with stress drops scaled as Benioff strain, and D) SR model with stress drop scaled as seismic moment.

Fortunately the means to estimate W, σ_{ij} and ϵ_{ij} at points in a volume for a faulting source exist in the form of algorithmns that calculate stress and strain due to a series of finite dislocations in an elastic halfspace (Okada, 1992) [7].

Ten overlapping finite dislocations are used to define a 4000 km long strike-slip fault (i.e., 10 times the longest earthquake source). I assume a "backslip" model for the loading configuration, i.e., dislocations with the opposite sense of slip to that of earthquake event (Savage and Prescott, 1978) [10]. The accumulated slip is assumed to be greatest at the surface and elliptically decreasing to zero at 15 km depth. W, σ_{ij} and ϵ_{ij} are calulated at the center of a grid of 1 km^3 blocks. The horizontal dimension of the grid scales with the size of the earthquake event (see below) but only extends to a depth of 20 km in the vertical dimension. Therefore the minimum grid size is 20 km x 20 km x 20 km.

To define the dimensions of the earthquake slip distributions a Brune source (Brune, 1970) [3] with a stress drop of 3 MPa was assumed and used to calculate the expected rupture area for earthquakes with M_w between 4 and 6. The empirical formulas of Wells and Coppersmith (1994) [13] were used to estimate rupture areas for strike-slip earthquakes with M_w between 5 and 8. These two means of estimating rupture area overlap very closely in the range $M_w = 5$ to 6, with both methods giving rupture areas of approximately 100 km^2 for an $M_w = 6$ earthquake. Therefore Brune source areas were used for $M_w \leq 6.0$ and Wells and Coppersmith (1994) [13] for $M_w > 6.0$.

As in the fault loading model, the earthquake source model consists of 10 overlapping finite dislocations. For rupture lengths less than 15 km, the earthquake source is rectangular (constrained by the algorithm) with slip greatest in the center and elliptically decreasing towards all edges. For rupture lengths larger than 15 km, the slip function mimics the loading function in the vertical dimension and also decreases elliptically towards the fault ends. The overall magnitude of slip is scaled to give the correct seismic moment for the correpsonding M_w.

In all cases I defined the magnitude of the loading to be at least 50 times that of the maximum earthquake slip. This, plus the smoothing nature of the overlapping dislocations with the total slip tapering to 0 at the edges, should prevent any "stress reversals" (at least in the shear stress) caused by edge effects in the model. Other loading models will need to be explored in future work.

Results

A series of models were constructed for earthquakes ranging from $M_w = 4.0$ to $M_w = 8.0$ (rupture lengths from 1 to 400 km) at $0.5M_w$ intervals. As described above, W was calculated on a grid scaled to the size of the earthquake rupture and difference in W between the loading model only and the loading plus earthquake rupture was taken as ΔE_{st}. The resulting scaling between M_w and ΔE_{st} is shown in Figure 2. Interestingly, strain energy release appears to scale close to $\log \Delta E_{st} = 2.2M_w$, not $1.5M_w$ as in seismic moment nor $0.75M_w$ as in Benioff strain. The overall magnitude of ΔE_{st} lies between seismic moment and Benioff strain.

The result above suggests that only the very largest earthquakes release significant amounts of the elastic potential energy stored in the seismogenic crust. Put into the context of the CA and SR results discussed earlier; it supports a CP over an SOC model for the earth's crust.

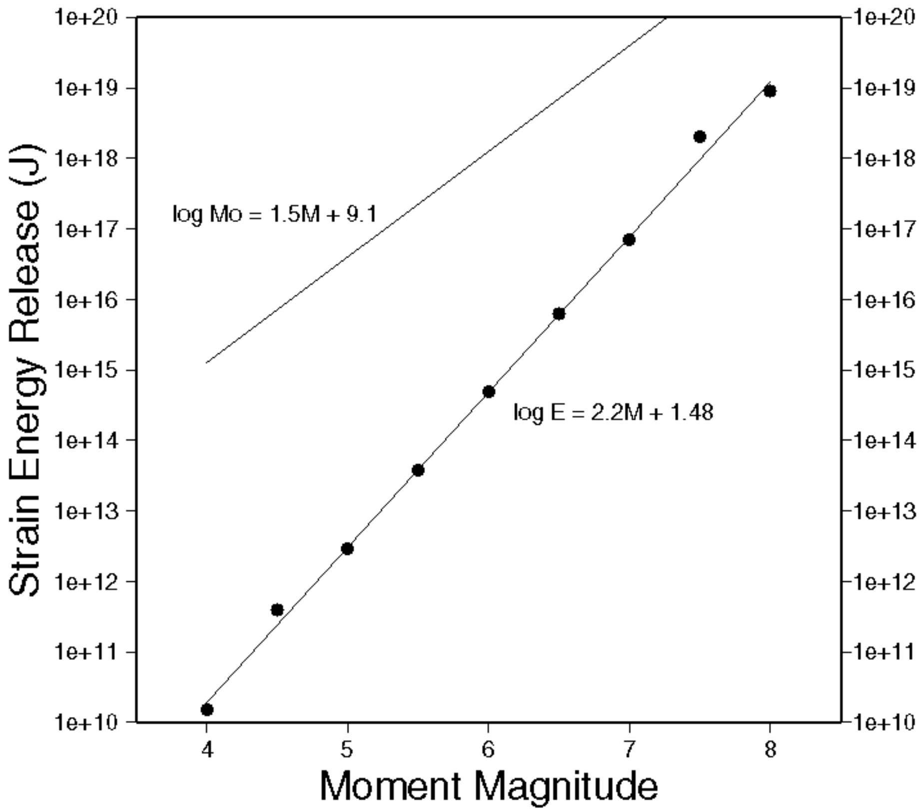

Figure 2: Strain energy release (Joules) versus moment magnitude for a model strike-slip fault (solid circles). A least squares regression line is also shown. Seismic moment versus moment magnitude is also shown for reference.

Discussion

It is clear from studies of earthquake rupture dynamics (e.g., Peyrat *et al.*, 2001) [8] that the pre-earthquake stress field on a fault is not likely to be smooth as it is in my assumed loading model. Also, some studies suggest that earthquakes may even dynamically rupture into areas that have little or no accumulated slip (Archuleta and Favreau, this volume) [2]. Archuleta and Favreau (this volume) [2] estimated an elastic energy release of $4.84 \times 10^{14} J$ for the 1979 $M_w = 6.5$ Imperial Valley strike-slip earthquake. This is an order of magnitude less than that predicted from Figure 2. It is probable that the estimates of strain energy release made here may form an upper bound. Further modeling efforts, particularly with different loading models, need to be explored.

Given that the estimated strain energy release has a different scaling relationship and in magnitude lies between those used in previous SR models of regional seismicity, it is unclear how SR models using the energy release scaling relationship estimated here would fare. If it is the energy release scaling that predominantly controls the dynamics, then CP-type behavior would be expected. If, however, the overall magnitude of the energy release is more important then a mixture of SOC and CP-behavior may result. New SR modeling efforts are required to sort these competing factors out.

Acknowledgments

Participation in the 3rd ACES International Workshop was supported by the Southern California Earthquake Center and the College of Charleston.

References

[1] Aki, K., and Richards, P.G. 1980, *Quantitative Seismology,* W.H. Freem and and Company, New York, 932 pp.

[2] Archuleta, R.J., and Favreau, P. 2002, *Seismic Energy Computed from Dynamic Models,* 3rd ACES International Workshop.

[3] Brune, J. 1970, *Tectonic Stress and the Spectra of Seismic Shear Waves from Earthquakes,* J. Geophys. Res., **75,** 4997–5009.

[4] Jaeger, J.C., and Cook, N.G.W. 1976, *Fundamentals of Rock Mechanics,* CHapman and Hall, New York, 585 pp.

[5] Jaumé, S.C., and Bebbington, M.S. 2000, *Accelerating Seismic Moment Release from Modified Stress Release Models (abstract),* EOS, 2000 Fall Mtg. Suppl **81,** F582.

[6] Jaumé, S.C., 2000, *Changes in Earthquake Size-Frequency Distributions Underlying Accelerating Seismic Moment/Energy Release,* In Geocomplexity and the Physics of Earthquakes, Geophysical Monogaph **120,** AGU, 199–210.

[7] Okada, Y., 1992, *Internal Deformation Due to Shear and Tensile Faults in a Half Space,* Bull Seism. Soc. Amer., **82,** 1018–1040.

[8] Peyrat, S., Olsen, K., and Madariaga, R., 2001, *Dynamic Modeling of the 1992 Landers Earthquake,* J. Geophys. Res., **106,** 26467–26482.

[9] Rundle, J.B., Klein, W. and Gross, S., 1999, *Physical Basis for Statistical Patterns in Complex Earthquake Populations: Models, Predictions and Tests,* Pure Appl. Geoph., **155,** 575–607.

[10] Savage, J.C. and Prescott, W.H., 1978, *Asthenosphere Readjustment and the Earthquake Cycle,* J. Geophys. Res., **83,** 3369–3376.

[11] Vere-Jones, D., Robinson, R. and Yang, W., 2001, *Remarks on the Accelerated Moment Release Model: Problems of Model Formulation, Simulation and Estimation,* Geophys. J. Int., **144,** 517–531.

[12] Weatherley, D., Jaumè, S.C. and Mora, P., 2000, *Evolution of Stress Deficit and Changing Rates of Seismicity in Cellular Automaton Models of Earthquake Faults,* Pure Appl. Geoph., **157,** 2183–2207.

[13] Wells, D.L., and Coppersmith, K.J., 1994, *New Empirical Relationships among Magnitude, Rupture Length, Rupture Width, Rupture Area, and Surface Displacements,* Bull. Seism. Soc. Amer., **84,** 974–1002.