A constitutive scaling law for the shear rupture that is inherently scale-dependent and physical scaling of nucleation time to dynamic instability

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The earthquake rupture that occurs in the Earth's crust characterized by inhomogeneities is a mixed process between what is called frictional slip failure and fracture of initially intact rock, and therefore, the governing law for earthquake ruptures must be a unifying law that governs both frictional slip failure and shear fracture. In addition, rupture phenomena including earthquakes are inherently scale-dependent, so that the governing law must also be formulated so as to scale scale-dependent physical quantities inherent in the rupture. These two requirements can be met, if the governing law is formulated as a slip-dependent constitutive law, and if geometric irregularity of the rupturing surfaces is properly incorporated into the law. The slip-dependent constitutive law is a unifying law that governs both frictional slip failure and shear fracture. Both slip-dependent constitutive formulation and incorporation of the geometric irregularity into the law are the key to physical scaling of scale-dependent physical quantities inherent in the shear rupture.

It has been shown that scale-dependent physical quantities inherent in the rupture are quantitatively scaled in the framework of fracture mechanics based on the slip-dependent formulation. The constitutive law parameters τ_p, $\Delta\tau_b$, and D_c are constrained by the following scaling relation:

$$D_c = a(\Delta\tau_b / \tau_p)^b \lambda_c \tag{1}$$

where a and b are numerical constants (a=0.662, and b=0.833 for granite rock in the brittle regime), and λ_c denotes the characteristic length representing a predominant wavelength component of geometric irregularity (roughness) of the rupturing surfaces. The constitutive scaling relation (1) makes it possible to scale scale-dependent physical quantities inherent in the rupture, such as the (apparent) fracture energy, the nucleation zone size, the breakdown zone size, and the slip acceleration. The nucleation time to dynamic instability is also consistently scaled in quantitative terms in the same framework. The length L of rupture nucleation increases up to the critical length L_c with time t according to a power law of the form:

$$L = L_c \left(\frac{t_a - t_c}{t_a - t} \right)^{1/(n-1)} = L_c \left(\frac{t_a - t_c}{t_a - t_c - (t - t_c)} \right)^{1/(n-1)} \qquad (t \le t_c) \tag{2}$$

where

$$t_a = \frac{1}{\alpha(n-1)} \frac{L_c}{V_s} \left(\frac{\lambda_c}{L_c} \right)^n + t_c \tag{3}$$

In the above equations, t_c is the critical time at which $L = L_c$ is attained, α and n are numerical constants ($\alpha = 8.87 \times 10^{-29}$, and $n = 7.31$), and V_s is the shear wave velocity. Since the nucleation time t scales with λ_c, it follows that the nucleation time to dynamic instability, $t - t_c$, is scaled completely by defining the dimensionless time T by $T=t/(\lambda_c/V_s)$. A universal scaling relation between L/λ_c and $T-T_c$ $(T_c=t_c/(\lambda_c/V_s))$ has been derived from (2), and this scaling relation is capable of unifying not only laboratory data on slip-failure nucleation under different conditions, but also field data on earthquake rupture nucleation. The present result leads to the same, previous conclusion that large earthquakes are in principle predictable.