# Ergodicity in Natural Fault Systems

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## Abstract

**Attempts to understand the physics of earthquakes over the past decade generally have focused on applying methods and theories developed based upon phase transitions, materials science, and percolation theory to a variety of numerical simulations of extended fault networks. These analyses attempt to model and explain many of the important features of the fault system, such as the scaling properties of the system, the variety of periodic, quasi-periodic, and nonperiodic behaviors, and the space-time clustering of events. Recent work suggests that the fault system can be interpreted as mean-field threshold systems in metastable equilibrium (Rundle et al., 1995; Klein et al., 1997; Ferguson et al., 1999), and that these results strongly support the view that seismic activity is highly correlated across many space and time scales within large volumes of the earth's crust (Rundle et al., 2000; Tiampo et al., 2002). In these systems, the time averaged elastic energy of the system fluctuates around a constant value for some period of time. These periods are punctuated by major events that reorder the system before it settles into another metastable energy well. One way to measure the stability of such a system is to check a quantity called the Thirumalai-Mountain (TM) energy metric (Thirumalai & Mountain, 1993; Klein et al., 1996). In particular, using this metric, we show that the actual California fault system is ergodic in space and time for the period in question, punctuated by the occurrence of large earthquakes.**

## Introduction

Recent work in the study of nonequilibrium systems, using computer simulations of simplified natural systems, suggests that certain equilibrium-like properties may be recovered at the appropriate spatiotemporal scales. In particular, numerical simulations of driven, mean-field systems suggest that far-from-equilibrium dissipative systems may be in a state of metastable equilibrium that can be characterized using equilibrium statistical mechanics (Rundle et al., 1995; Klein et al., 1996; Ferguson et al., 1999; Egolf, 2000). To date, however, there has been no definitive evidence that the same principles can be applied to the natural systems these simulations are expected to reproduce. Here we show that the natural earthquake fault system displays ergodicity in its activity in order to demonstrate that it, like the slider block simulations models, is a mean-field system in a state of metastable equilibrium.

## Background

Threshold systems are known to be some of the most important nonlinear, self-organizing systems in nature, including networks of earthquake faults, neural networks, superconductors and semiconductors, and the World Wide Web, as well as many political, social, and ecological systems. All of these systems have dynamics that are strongly correlated in space and time, and all typically display a multiplicity of spatial and temporal scales. In particular, if the range of interactions between elements of the system is long and weak, so that the dynamics can be understood as mean-field, fluctuations tend to be suppressed and the system may approach a stationary state (Kac et al., 1961; Gaspard et al., 1998). In addition, Boltzmann fluctuations, which are an important property of equilibrium systems, were directly observed in driven mean-field slider block simulations (Rundle et al., 1995; Morein et al. 1997; Klein et al., 2000; Main, et al. 2000; Rundle et al., 2002). Recently, Ferguson (1999) demonstrated that an energy fluctuation metric (Thirumalai et al, 1989; Thirumalai and Mountain, 1993) originally developed to test for the presence of ergodic behavior in equilibrium systems showed that driven mean-field slider block models could also be considered to demonstrate ergodic behavior over finite intervals of time. Finally, direct observations of Gaussian fluctuations and detailed balance in transition probabilities suggest ergodic behavior in a driven system of mean-field coupled map lattices (Egolf, 2000).

The critical question is whether these conclusions from model driven systems can be extended to natural driven systems. In particular, we might consider as candidates the class of driven threshold systems, of which the slider block models are one case. Earthquake fault systems are one example of such a complex nonlinear driven threshold system. As in other threshold systems, interactions among a spatial network of fault segments are mediated by means of a potential that, in this case, allows stresses to be redistributed to other segments following slip on any particular segment. For faults embedded in a linear elastic host, this potential is a stress Green's function whose exact form is calculated from the equations of linear elasticity, once the geometry of the fault system is specified. A persistent driving force, arising from plate tectonic motions, increases stress on the fault segments (Rundle et al., 1995; Morein et al. 1997; Klein et al., 2000).

Once the stresses reach a threshold characterizing the limit of stability of the fault, a sudden slip event results. The slipping segment can also trigger slip at other locations on the fault surface whose stress levels are near the failure threshold as the event begins. In this manner, earthquakes result from the interactions and nonlinear nature of the stress thresholds (Huang, et al., 1998; Rundle et al., 1999; Klein et al., 2000).

Finally, in the mean-field regime, as the interaction length becomes large, leading to a damping of the fluctuations, a mean-field spinodal appears that is the classical limit of stability of a spatially extended system (Penrose and Lieberman, 1976; Gopal and Durian, 1995). Examined in this limit, driven threshold systems appear to be locally ergodic, and display equilibrium behavior when driven at a uniform rate. Following the initial discovery that driven mean-field slider block systems with microscopic noise display equilibrium properties (Rundle et al., 1995), other studies have confirmed local ergodicity, the existence of Boltzmann fluctuations in both these and other mean- or near mean-field systems, and the appearance of an energy landscape, similar to other equilibrium systems (Klein et al., 1996; Ferguson et al., 1999; Main, et al. 2000; Rundle et al., 2002). Thus the origin of the physics of scaling, critical phenomena and nucleation appears to lie, at least in part, in the ergodic properties of these mean-field systems.

Note that the spatial and temporal firing patterns of such driven threshold systems are complex and often difficult to understand and interpret from a deterministic perspective, as these patterns are emergent processes that develop from the obscure underlying structures, parameters, and dynamics of a multidimensional nonlinear system (Nijhout, 1997). For example, in the earth, there is no means at present to measure the stress and strain at every point in an earthquake fault system, or the constitutive parameters that characterize this heterogeneous medium and its dynamics.

However, the seismicity, the firing patterns that are the surface expression or proxy for the dynamical state of the underlying fault system, can be located in both space and time with considerable accuracy (Bakun and McEvilly, 1984; Sieh et al., 1989; Hill et al., 1990). If this natural system is also, as simulations suggest, a mean-field threshold system in metastable equilibrium (Klein et al., 1997), then the time averaged elastic energy of the system fluctuates around a constant value for some period of time. These periods are punctuated by major events that reorder the system before it settles into another metastable energy well. Here we wish to compare the ergodic properties of the simulated and natural systems in terms of their energy release, an easily measured quantity. We employ a quantity called the Thirumalai-Mountain (TM) energy fluctuation metric in order to make this comparison (Thirumalai et al, 1989; Thirumalai and Mountain, 1993; Klein et al., 1996; Ferguson et al., 1999).

## Method

The TM metric measures effective ergodicity, the difference between the time average of a quantity, generally related to the energy of the system, and the ensemble average of that same quantity over the entire system. This metric measures effective ergodicity, the difference between the time average of the quantity $E_j$ at each site, and the ensemble average of the entire system, because the derivation of the fundamental equations requires the equivalence of the time and ensemble averages of the E-fluctuating quantities. The fundamental idea is that of statistical symmetry, in which the N oscillators, particles, cells, or spins in the system are statistically identical, in terms of its averaged properties (Thirumalai et al, 1989; Thirumalai and Mountain, 1993).

While, in general, a system is ergodic for infinite averaging times, if the actual measurement time scales are finite, but long, all regions of phase space are sampled with equal likelihood, and the system is effectively ergodic (Thirumalai et al, 1989).

Statistically, ergodicity means that, over a large enough representative sample in time and space, the spatial and temporal averages are constant. Ergodicity is a behavior that is limited to equilibrium states, in which transition probabilities are univarying or follow a definite cycle, and implies stationarity as well. Note that if such a system is ergodic, it is also in metastable equilibrium and can be analyzed as such.

A system is said to be effectively ergodic if, for a given time interval, the system has equivalent time-averaged and ensemble-averaged properties (Palmer, 1982; Thirumalai et al, 1989; Thirumalai and Mountain, 1993). The energy-fluctuation metric $\Omega_e(t)$, proposed by Thirumalai and Mountain is

$$\Omega_e(t) = \frac{1}{N}\sum_{i=1}^{N}[\varepsilon_i(t) - \bar{\varepsilon}(t)]^2, \tag{1}$$

where

$$\varepsilon_i(t) = \frac{1}{t}\int_0^t E_i(t')dt' \tag{2}$$

is the time average of a particular individual property, , related to the energy of the system, and

$$\bar{\varepsilon}(t) = \frac{1}{N}\sum_{i}^{N}\varepsilon_i(t) \tag{3}$$

is the ensemble average over the entire system. If the system is effectively ergodic at long times, $\Omega_e(t) = \frac{D}{t}$, where $D$ is a constant that measures the rate of ergodic convergence (Thirumalai et al, 1989; Thirumalai and Mountain, 1993). A more physical way of thinking about the fluctuation metric is to consider that the oscillator has been mapped into a Brownian particle, and equation (1) is therefore an expression of the equivalence between the time averages of particle behavior, and the ensemble averages over particle states after a series of Brownian increments.

In slider block models used to replicate the behavior of earthquake fault networks, as the interaction range increases, the system approaches mean-field limit behavior. If, as a result, these slider block models are in metastable equilibrium, they can be analyzed using the methods and principles of statistical mechanics. Ferguson (1999) applied the TM metric to slider block numerical simulations in order to show that the system was ergodic at external velocities, $V$, that approach $V = 0$, and retrieved a linear relationship between the inverse TM metric and time, denoting effective ergodicity as defined above (Klein et al., 1996; Ferguson et al., 1999).

The question that remains to be answered is whether the same applies to natural earthquake fault networks. Interactions in a natural driven system should be mean-field if the results on ergodicity are to hold. Since slider block models were originally conceived as models of earthquake faults, it is therefore logical to investigate the presence or absence of ergodic behavior in systems of earthquake faults. We proceed to test this hypothesis using the TM fluctuation metric.

Here we apply the TM metric to the surface expression of the underlying energy landscape, the seismicity in a regional fault system, southern California. For application to the earthquake fault system, the number of earthquakes of a particular magnitude or greater can be expressed as a function of the seismic energy release (Kanamori, 1977; Turcotte, 1997). Therefore, in this calculation of the TM metric, $E_i(t) \equiv R_i(t)$, the number of events greater than magnitude three in southern California in each year.

The seismicity data employed in our analysis is taken from existing observations in southern California between the years 1932 and the present. Using only the subset of this data at locations **x** in southern California and covering the period from January 1, 1932 through December 31, 2001, we compute the TM metric for southern California seismicity, over the region 32º to 40º latitude, -115º to –125º longitude. Note that we use only events having magnitude $M \geq 3$, to ensure completeness of the catalog, and the catalog is not declustered in any way.

## Results

In Figure 2, we plot the inverse TM energy metric for the variance in number of events in southern California over time, from 1932 through 2001. Note, again, the linear relationship between the inverse TM metric and time. The natural fault system appears to be effectively ergodic for relatively long periods of time, punctuated by the occurrence of large earthquakes, such as the Kern County event of 1952, the Imperial Valley earthquake in 1979, the Landers sequence of 1991, and the Hector Mine earthquake in 1999. We interpret these events in terms of a physical picture where the fault system resides for long periods in an ergodic, local energy minimum on a complex energy landscape. Occasionally, however, the nonlinear dynamics leads to an earthquake, which has the effect of causing the system to suddenly depart from its local energy minimum, and to migrate to a new local minimum, where it again resides in an effectively ergodic state.

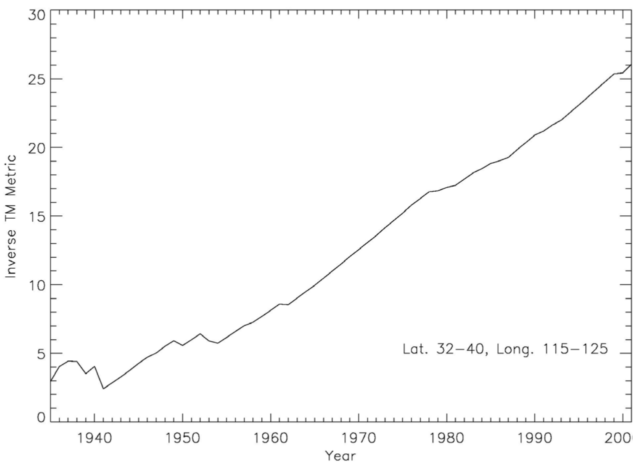


Figure 1: The inverse TM metric for seismicity numbers plotted versus time, for southern California. Note the periods of ergodic behavior, punctuated by the occurrence of large events.

## Conclusions

In summary, the observable properties of self-organizing, strongly correlated, near mean-field driven threshold systems arise from the underlying, locally ergodic dynamics. We employ the well-known Thirumalia-Mountain fluctuation metric to identify the presence of one equilibrium property in the dynamics of the natural earthquake fault system in California, ergodicity, using data from existing seismic monitoring networks, as numerical simulations have demonstrated that the dynamics of driven mean-field systems of interacting slider blocks and coupled map lattices display strong evidence of ergodic behavior. Our results suggest that this natural system is effectively ergodic, in metastable wells for periods of time on the order of several decades, and, therefore, mean-field as in the numerical simulations that are used to study these systems, displaying critical point behavior with correlations over a range of spatial and temporal scales. This also suggests that many of the observed properties of the natural system, such as Gutenberg-Richter scaling, large correlation lengths, and the classification of earthquakes as nonclassical nucleation events, can be understood as manifestations of an effectively ergodic, nonlinear system. As the dynamical system evolves, migration to a new energy minimum occurs in association with the occurrence of a large earthquake. Finally, this supports the previous work using numerical simulations and validates their use in the study of natural, driven threshold systems in general and earthquake fault systems in particular.

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## References

[1] Bakun, W.H., T.V. McEvilly, 1984, J. Geophys. Res., **89**, 3051.

[2] Egolf, D., 2000, Science, **287**, 101.

[3] Ferguson, C.D., W. Klein, J.B. Rundle, 1999, Phys. Rev. E, **60**, 1359.

[4] Gaspard, P., M.E. Briggs, M.K., Francis, et al., 1998, Nature, **394**, 865.

[5] Gopal, A.D., D.J. Durian, 1995, Phys. Rev. Lett., **75**, 2610.

[6] Hill, D., J.P. Eaton, L.M. Jones, USGS Prof. Paper 1515, (U.S. GPO, Washington, D.C., 1990) pp. 115-152.

[7] Hopfield, J.J., 1982, Proc. Nat. Acad. Sci. U.S.A., **79**, 2554.

[8] Huang, Y., H. Saleur, C. Sammis, D. Sornette, 1998, Europhys. Lett., **41**, 43.

[9] Kac, M., G.E. Uhlenbeck, P.C.Hemmer, 1961, J. Math.Phys,. **4**, 216.

[10] Kanamori, H., 1977, J. Geophys. Res., **82**, 2981.

[11] Klein, W., M. Anghel, C.D. Ferguson, J.B. Rundle, J. Sá Martins, in *Geocomplexity & the Physics of Earthquakes*, Geophysical Monograph Ser. **120,** J.B. Rundle, D.L. Turcotte, W. Klein, Eds. (AGU, Washington D.C., 2000) pp. 43-71.

[12] Klein, W., C. Ferguson, J.B. Rundle, 1996, in *Reduction and Predictability of Natural Disasters, SFI series in the science of complexity, XXV*, J.B. Rundle, D.L. Turcotte, W. Klein, Eds., (Addison-Wesley, Reading, MA, 1996), 223-242.

[13] Klein, W., J.B. Rundle, C.D. Ferguson, 1997, Phys. Rev. Ltrs., **78**, 3793.

[14] Main, I.G., G. O'Brien, J.R. Henderson, 2000, J. Geophys. Res., **105**, 6105.

[15] Morein, G., D.L. Turcotte, A. Gabrielov, 1997, Geophys. J. Int., **131**, 552.

[16] Nijhout, H.F., in *Pattern Formation in the Physical and Biological Sciences, Lecture Notes V, SFI,* (Addison Wesley, Reading, MA, 1997) pp. 269-298.

[17] Palmer, R.G., 1982, Adv. Phys., **31**, 669.

[18] Penrose, O., J.L. Leibowitz, in *Fluctuation Phenomena*, E.W. Montroll, J.L. Leibowitz, Eds. (North-Holl, Amsterdam, 1976) pp. 323-375.

[19] Rundle, J.B., W. Klein, S. Gross, 1999, Pure Appl. Geoph., **155**, 575.

[20] Rundle, J.B., W. Klein, S. Gross, D.L. Turcotte, 1995, Phys. Rev. Lett., **75**, 1658

[21] Rundle, J.B., W. Klein, K.F. Tiampo, J. Sá Martins, 2002, Proc. Nat. Acad. Sci. U.S.A., Suppl. 1, **99**, 2463.

[22] Sieh, K., M. Stuiver, D. Brillinger, 1989, J. Geophys. Res., **94**, 603.

[23] Thirumalai, D., R.D. Mountain, 1993, Phys. Rev. E, **47**, 479.

[24] Thirumalai, D., R.D. Mountain, T.R. Kirkpatrick, 1989, Phys. Rev. A, **39**, 3563.

[25] Tiampo, K.F., J.B. Rundle, S.J. Gross, S. McGinnis, S., and W. Klein, 2002, *J. Geophys. Res.*, accepted for publication.

[26] Turcotte, D.L., *Fractals and chaos in geology and geophysics,* 2nd ed. (Cambridge University Press, Cambridge, U.K., 1997).