Accelerating Precursory Activity in Statistical Fractal Automata

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Abstract

A statistical fractal automaton model is described which displays two modes of dynamical behaviour. The first mode, termed *recurrent criticality*, is characterised by quasi-periodic, characteristic events which are preceded by accelerating precursory activity. The second mode is more reminiscent of SOC automata in which large events are not preceded by an acceleration in activity. Extending upon previous studies of statistical fractal automata, a redistribution law is introduced which incorporates two model parameters: a dissipation factor and a stress transfer ratio. Results from a parameter space investigation show that a straight line through parameter space marks a transition from recurrent criticality to unpredictable dynamics. Recurrent criticality only occurs for models within one corner of parameter space. The location of the transition displays a simple dependance upon the fractal dimension of the cell strength distribution.

Introduction

Both the Gutenberg-Richter frequency-magnitude relation of earthquakes (Gutenberg and Richter, 1954) and the Bufe-Varnes time-to-failure relation describing accelerating precursory activity (Bufe and Varnes, 1993), have been cited as evidence that seismicity bears some resemblence to systems operating at or near a Critical Point. Such systems have been studied for a number of decades by Statistical Physicists. Cross-pollination of ideas has led to two related yet distinct viewpoints of seismicity. The concept of self-organised criticality (SOC) provides an explanation for the dynamical origins of the GR relation (Bak and Tang, 1989). Accelerating precursory activity has been identified as a signature of a system approaching a Critical Point (Sornette and Sammis, 1995).

An explanation for both the GR relation and accelerating precursory activity may involve a synthesis of these two viewpoints. It is hypothesised that the Earth's crust is a system which naturally self-organises towards a Critical Point (as in SOC systems) however the largest events perturb the system away from the critical state. Subsequent to such an event, the system once again self-organises towards criticality (and another large event). Accelerating precursory activity occurs each time the system approaches criticality. In the following we refer to such behaviour as *recurrent criticality*.

Recurrent criticality has been identified in automata employing a geometrical fractal distribution of cell sizes (Huang *et al.* ,1998; Sammis and Smith, 1999). From a comparative analysis of SOC models and geometrical fractal automata (Sammis and Smith, 1999), it was concluded that the largest events in recurrent criticality models, must fail a significant portion of the system and dissipate an appreciable amount of energy from the system, in order to perturb the system from criticality. Weatherley *et al.* (2000) identified recurrent criticality in a nearest neighbour automaton employing a statistical fractal disribution of cell strengths rather than a fractal distribution of cell sizes. This model was compared with another statistical fractal automaton which did not display recurrent criticality. The only difference between the two models was in the laws for redistribution of stress from failed cells. The current research extends upon these results.

A stress redistribution law is introduced which is a generalisation of the two laws employed in the previous statistical fractal automata. The generalised redistribution rule involves two model parameters: a dissipation factor determining the fraction of stress dissipated from failed cells and, a stress transfer ratio determining the fraction of stress redistributed to previously failed or unfailed nearest neighbours. In the following, the new model is briefly described and results from parameter space investigations are presented. The main conclusion is that this model exhibits a sharp transition between recurrent criticality and behaviour more reminiscent of SOC models. The transition corresponds to a straight line across one corner of parameter space. There is a systematic dependence of the location of the transition line, upon the fractal dimension of the cell strength distribution.

Model Description

The models consist of a rectangular grid of 128×128 cells, each of which is assigned a constant, scalar strength and a variable, scalar stress. The strengths of cells are pre-defined according to a statistical fractal distribution which is generated using a Fourier filtering technique. Initially, the stresses of all cells are set to zero. External loading is achieved by periodically incrementing the stress of all cells by an amount $\Delta\sigma$, computed dynamically as the minimum stress increment necessary to fail at least one cell. A cell fails when its stress equals or exceeds the strength of that cell.

The two statistical fractal automata examined by Weatherley *et al.* (2000) differed only in the redistribution laws employed to transfer stress from failed cells to the nearest neighbours. Recurrent criticality was identified in a model which transferred all of the stress of failed cells only to nearest neighbours which had not previously failed in the current rupture cascade. Unpredictable, SOC-like dynamics was identified in the other model, in which a proportion of the stress of failed cells was transferred to previously failed neighbours.

This difference motivated the derivation of a new redistribution law in which the ratio of stress transferred to previously failed neighbours compared with the stress transferred to previously unfailed neighbours, is given by a model parameter (κ) termed the stress transfer ratio. In addition to this new redistribution law, circular (rather than dissipative) boundaries are employed in the new models. A dissipation factor (γ) controls the amount of stress dissipated from failed cells prior to redistribution of stress to the four nearest neighbours.

If a failed cell supports a stress of σ_f and n_b of its four nearest neighbours have previously failed since the last stress increment was applied, the stress transferred to previously unbroken or previously broken nearest neighbours is given by:

$$\Delta\sigma_u = \frac{\gamma\sigma_i}{[4-(1-\kappa)n_b]} \tag{1}$$

$$\Delta\sigma_b = \frac{\kappa\gamma\sigma_i}{[4-(1-\kappa)n_b]} \tag{2}$$

where $\Delta\sigma_u$ is the amount of stress transferred to unbroken neighbours and $\Delta\sigma_b$ is the stress transferred to broken neighbours. Note that by definition, $\frac{\Delta\sigma_b}{\Delta\sigma_u} = \kappa$, the stress transfer ratio.

Results from parameter space investigations

Initially a parameter space investigation was performed in which κ and γ were varied while the fractal dimension of the strength distribution was held fixed at $D = 2.3$. The range of cell strengths was chosen to be $[0.1, 1.0]$. Some preliminary simulations indicated that the results presented are relatively independent of the range of strengths employed. A total of 399 simulations were performed, employing 21 values of κ in the range $0 \leq \kappa \leq 1$ and 19 values of γ in the range $0.05 \leq \gamma \leq 0.95$. For each pair of parameter values (κ, γ) a simulation consisting of 5×10^5 loading timesteps was performed.

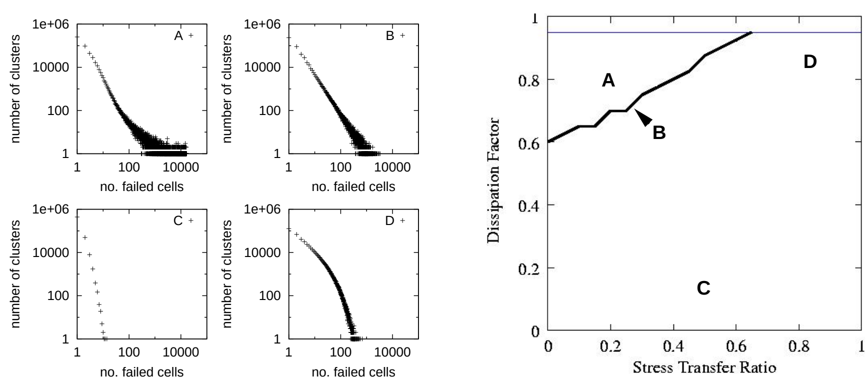

Figure 1: LEFT: Examples of the four classes of event distributions. RIGHT: The relative locations of each class. The thick line corresponds with simulations displaying class B, power-law scaling for all event sizes.

An interval event size distribution, employing the number of failed cells as a measure of event size, was produced for each simulation. The 399 event size distributions were then visually compared one with another. Four different classes of event distribution were identified, corresponding to different regions of parameter space. These four classes are illustrated in Figure 1. The regime of parameter space corresponding to each class is also indicated. The thick line joins the parameters of models displaying power-law scaling for all event sizes. In the corner of parameter space corresponding to small values of κ and relatively small amounts of dissipation (larger values of γ), the event distribution has an overabundance of larger, characteristic events. In the rest of parameter space, the event distribution rolls-over either for the large event tail of the distribution, or rolls over for all event sizes.

Time-series of mean-stress were also analysed (see Figure 2). Large, saw-tooth fluctuations in mean stress qualitatively similar to those obtained in geometrical fractal automata by Sammis and Smith (1999), were obtained for simulations with a charactistic event distribution. The saw-tooth fluctuations displayed quasi-periodicity and large drops in mean stress corresponded to the larger, characteristic events. Along the line of GR-scaling simulations, mean stress fluctuations are considerably smaller in amplitude and more irregular. Simulations with either type of roll-over distribution displayed highly irregular, very small amplitude fluctuations. To summarise the variation in mean stress fluctuations throughout parameter space, a surface plot (Figure 2 was produced indicating the time-averaged mean stress and the maximum and minimum mean stress for each simulation (computed once the initial transient was completed). The fold at which the time-averaged mean stress drops significantly below the rest of parameter space corresponds reasonably well with the line of power-law scaling simulations identified by the event distributions.

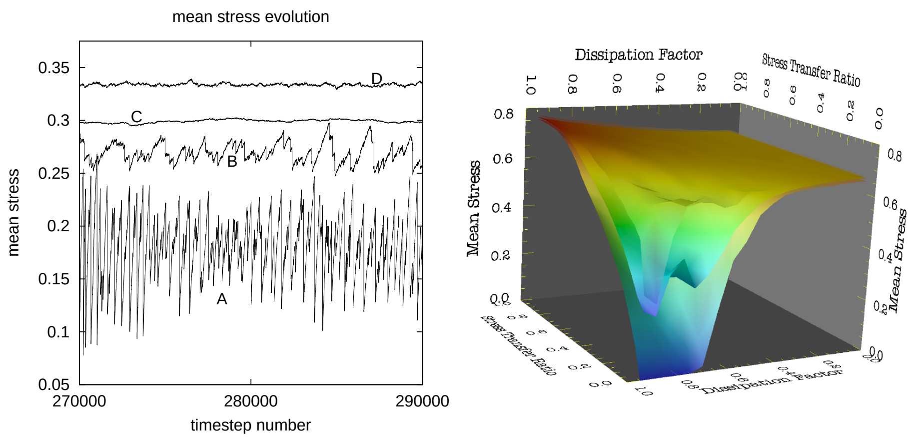

Figure 2: LEFT: Typical mean stress time-series corresponding to each class of event distributions. RIGHT: Parameter space plot of the time-averaged mean stress and range of mean stress fluctuations for each simulation.

A time-to-failure analysis demonstrated that accelerating precursory activity only occurs for models with a characteristic event distribution. The cumulative stress release time-series prior to the 100 largest events from each simulation, were selected for analysis. For each large event, the Bufe-Varnes relation (without log-periodic corrections) was fitted to the cumulative stress release within a variable time-window, preceeding the large event. The power-law exponent (c) of the Bufe-Varnes law was also varied. The best power-law fit was selected for each large event by determining which fit minimised a goodness-of-fit parameter ($\mathcal{C}$), defined as the ratio of the r.m.s. error of the power-law fit to the r.m.s error of a linear fit. A goodness-of-fit parameter, $\mathcal{C}< 0.7$ and an exponent value, $c < 0.8$ were the conditions used to determine whether the selected power-law fit represented a reasonable acceleration in precursory activity.

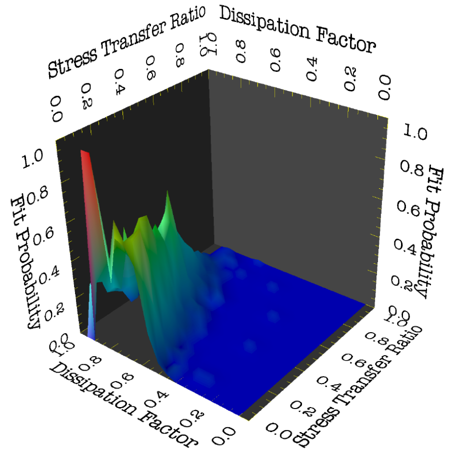

Figure 3: Parameter space plot of the probability that the largest events in each simulation are preceded by accelerating precursory activity.

For each simulation, we computed the probability that a large event is preceeded by a reasonable acceleration in activity. This fit probability is plotted as a parameter space surface plot in Figure 3. Only simulations within the characteristic event corner of parameter space were found to have a significant probability of accelerating precursory activity. The transition to a constant rate of activity is clearly emphasised and corresponds well with the line of GR-scaling simulations. We concluded from the above results that the line of GR-scaling simulations marks a transition from recurrent criticality (in one corner of parameter space) to unpredictable dynamics which bear more resemblence to the dynamics of SOC automata. Intermediate-term forecasting of the largest events is achievable only in models displaying recurrent criticality.

Three more parameter space studies have been performed using different values of the fractal dimension of the strength distribution ($D = 1.875, 2.25, 2.625$). These parameter space studies concentrated upon the region of parameter space surrounding the transition line. For each fractal dimension, over 800 simulations were performed and event distributions were generated. Visual examination of the event distributions permitted the identification of the transition line for each fractal dimension, at a higher resolution than in the original parameter space study.

Figure 4 contains a plot of the approximate location of the transition line for each fractal dimension. The results suggest that the transition line is best-approximated by a straight line through parameter space. The slope of the line appears to be independent of the fractal dimension, however its location in parameter space varies with the fractal dimension. The lower the fractal dimension (corresponding to predominantly longer-range clustering in the strength distribution) the larger the recurrent criticality regime in parameter space. Conversely, the larger the fractal dimension (corresponding to a more random strength distribution) the smaller the recurrent criticality regime.

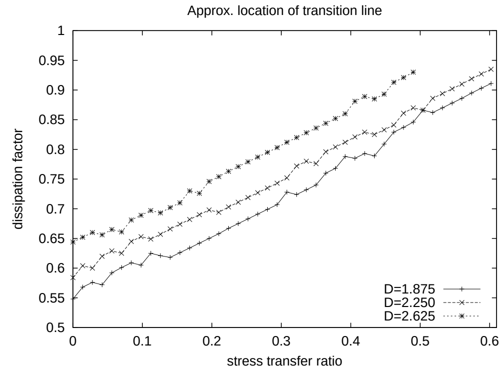

Figure 4: Approximate location of the transition line in parameter space for three parameter space investigations employing different fractal dimensions of the cell strength distribution.

Acknowledgments

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