Critical Sensitivity in Driven Nonlinear Threshold Systems

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Abstract

An analysis based on driven nonlinear threshold model with damage relaxation time and mean field approximation shows that critical sensitivity might be a common precursory feature of catastrophe in heterogeneous brittle media. So, it may be a helpful clue to prediction and provides clear warning. The critical sensitivity is applied to our rupture experiments on rocks.

Introduction

A great earthquake may be thought as rupture in heterogeneous brittle media. The rupture displays complexity ([1-9]), in particular, catastrophe transition ([1,4,6]) and sample-specificity ([2,3]). The rupture appears to be a transition from globally stable accumulation of damage to catastrophic failure. Also, the catastrophe may be different from sample to sample for samples identical macroscopically. This leads to macroscopic uncertainty. It is insufficient to represent the catastrophe of a system by its macroscopically average properties only. These result in difficulty of prediction. Such a kind of complexity is associated with wide range of characteristic space and time scales in the concerned problems. Especially, the coupling between different scales and the coupling between disordered heterogeneity and dynamical nonlinearity play essential roles in the catastrophe transition.

The critical sensitivity means that a heterogeneous brittle medium may become significantly sensitive prior to its catastrophe transition point. We found that the critical sensitivity might be a common precursory feature of catastrophe, and may be a helpful clue to prediction of catastrophe ([5]). Our analysis is based on a driven nonlinear threshold model with damage relaxation time. The analytical approach adopted mean field approximation. In numerical simulation, stress fluctuations were taken into account. In addition, we made experiments on rock rupture and found that the critical sensitivity provides clear warning of rupture.

Physical Model

Consider a system comprised of a great number of interacting, mesoscopic units. The mesoscopic heterogeneity is described by prescribed threshold σ_c of each unit. The threshold follows a statistical distribution function $f(\sigma_c, t)$ with initial condition $f(\sigma_c, t=0) = h(\sigma_c)$, where $h(\sigma_c)$ is normalized. When a unit breaks, it will be excluded from the distribution function $f(\sigma_c, t)$. The real driving force σ on each unit is determined by nominal driving force σ_0 according to stress re-distribution model. The simplest model is based on mean field approximation, i.e., the nominal driving force σ_0 is supported uniformly by all intact units

$$\sigma = \frac{\sigma_0}{1-p}\ , \tag{1}$$

where p is damage fraction

$$p = 1 - \int_0^\infty f(\sigma_c, t) \mathrm{d}\sigma_c\,. \tag{2}$$

The statistical damage evolution is assumed to satisfy the following equation based on damage relaxation time model

$$\frac{\partial f(\sigma_c, t)}{\partial t} = -\frac{f(\sigma_c, t)}{\tau}, \tag{3}$$

where τ is the relaxation time of damage evolution. For simplicity, we assume

$$\tau = \begin{cases} \text{constant } \tau_D, & \text{as } \sigma \geq \sigma_c \\ \infty, & \text{as } \sigma < \sigma_c \end{cases} \tag{4}$$

Critical Sensitivity under Quasi-static Loading

Now, we consider the case of quasi-static and monotonous loading. In such a case, the equilibrium state can be reached for each loading step. The equilibrium damage fraction at σ_0 can be calculated from the following equation:

$$P(\sigma_0) = \int_0^{\frac{\sigma_0}{1-P(\sigma_0)}} h(\sigma_c) d\sigma_c\ . \tag{5}$$

The response of the system to increasing driving force can be defined by

$$R(\sigma_0)=\frac{\mathrm{d}E(\sigma_0)}{\mathrm{d}\sigma_0}\ , \tag{6}$$

where $E\{\sigma_0\}$ is the cumulative energy release. It is found that the damage evolution displays catastrophe transition at $\sigma_0=\sigma_{0\mathrm{f}}$, which is determined by equation

$$\frac{\mathrm{d}P(\sigma_0)}{\mathrm{d}\sigma_0}=\infty\,. \tag{7}$$

Below the transition point, damage fraction increases continuously, corresponding to stable cumulation of damage. However, at the transition point, the system falls into a situation of self-sustained catastrophic failure, then a finite jump of damage fraction occurs.

In order to measure the sensitivity of the response to nominal driving force, we define the sensitivity as

$$S(\sigma_0)=\frac{\sigma_0}{R(\sigma_0)}\frac{\mathrm{d}R(\sigma_0)}{\mathrm{d}\sigma_0}\ . \tag{8}$$

A highly sensitive state is characterized by $S>>1$.

We adopted the initial distribution function as Weibull distribution function

$$h=m\sigma_{\mathrm{c}}^{\ m-1}\exp\left(-\sigma_{\mathrm{c}}^{\ m}\right). \tag{9}$$

The cumulative energy release $E(\sigma_0)$ and sensitivity $S(\sigma_0)$ for globally mean field model (GMF) are presented in Figure 1 (for Weibull distribution function with modulus $m=3$). The cumulative energy release shows that the evolution displays a catastrophe transition at $\sigma_0=\sigma_{0\mathrm{f}}$. At the initial stage $\sigma_0\sim 0$, S keeps in a low level, corresponding to an insensitive state. However, we can see that $S\to\infty$ as $\sigma_0\to\sigma_{0\mathrm{f}}$, which implies that the system becomes sensitive significantly as it approaches to its catastrophe transition point. Such a feature is called critical sensitivity, which is an important precursor of catastrophe.

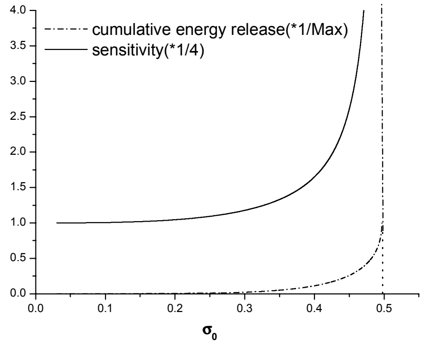

Figure 1: Critical sensitivity in driven nonlinear threshold systems for quasi-static and monotonous loading based on mean field approximation and damage relaxation time model. $h(\sigma_{\mathrm{c}})$ is Weibull distribution function with modulus m =3. σ_0 is nominal driving force and E is the cumulative energy release normalized by $E_{\mathrm{cum}} = \int_0^{\sigma_{0f}} R(\sigma_0)\, \mathrm{d}\sigma_0$. The cumulative energy release displays a catastrophe at $\sigma_0 = \sigma_{0f} = (me)^{-\frac{1}{m}}$, and $\sigma_{0f} = 0.4968$ for $m = 3$. The sensitivity S is defined by Equation (8) and normalized by $S(\sigma_0 = 0) = m + 1$. S increases significantly prior to catastrophe suggesting critical sensitivity.

Loading Rate Effect on Catastrophe and Critical Sensitivity

The assumption of quasi-static loading is valid only for the case that the characteristic time of loading is much longer than τ_{D}, the characteristic time of damage relaxation. Otherwise, the loading rate effect should be taken into account. Now we consider the loading rate effect. As an example, we assume that the nominal driving force increases with time linearly: $\sigma_0 = \alpha \frac{\sigma_{0\mathrm{f}}}{\tau_{\mathrm{D}}} t$, where α is a constant. Define time scale of loading by $\tau_0 = \frac{\tau_{\mathrm{D}}}{\alpha}$, which means that $\sigma_0 = \sigma_{0\mathrm{f}}$ as $t = \tau_0$. So, α is really the ratio the time scale of damage relaxation to that of loading. The sensitivity of response of the system to nominal driving force can be measured by

$$S(t) = \frac{\sigma_0(t)}{R(t)} \frac{\frac{\mathrm{d}R(t)}{\mathrm{d}t}}{\frac{\mathrm{d}\sigma_0(t)}{\mathrm{d}t}}, \tag{10}$$

where R is energy release rate. Figure 2 shows the damage rate, energy release rate and the sensitivity. In such a case, the main rupture, or catastrophic rupture, can be defined as the stage with high damage rate or great increase of energy release rate. We can see that the sensitivity increases significantly prior to catastrophic rupture, suggesting a critical sensitivity. In addition, the appearance of catastrophe and critical sensitivity are delayed by the loading rate effect to compare with that in the case of quasi-static loading.

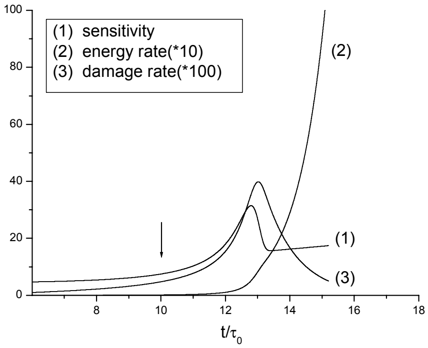

Figure 2: Critical sensitivity in the case with time-dependent nominal driving force $\sigma_0\left(t\right)=\alpha\frac{\sigma_{0\mathrm{f}}}{\tau_0}t$ with $\alpha=0.1$. The arrow indicates the catastrophe transition point in the case of quasi-static loading. The initial distribution function is Weibull distribution function with modulus $m=3$. The significant increase of sensitivity prior to the peak of damage rate or the great increase of energy release suggests critical sensitivity.

Effects of Stress Fluctuations on Catastrophe and Critical Sensitivity

In mean field approximation, stress fluctuations are neglected, however, the stress fluctuations play an essential role in catastrophe. As we take the stress fluctuations into account, the problem becomes much more complicated. Numerical results showed that the evolution also presents catastrophe transition but the catastrophe displays sample-specificity. Even though, it is also found that the critical sensitivity is really the common precursory feature of catastrophe in the case with stress fluctuations, see reference [5].

Critical Sensitivity in Rock Rupture Experiments

Now we show some results of rock rupture experiments ([10]) on samples 1050*400*100 mm^3 (for marble and gneiss) and 1050*400*150 mm^3 (for sandstone). The samples were compressed until rupture. The energy release was recorded by acoustic emission. The sensitivity is defined by

$$S = \frac{F}{R} \frac{\Delta R}{\Delta F}, \tag{11}$$

where F is external load, R is energy release rate, and ΔR is the increment of energy release rate corresponding to the increment of external load ΔF. The experimental results are shown in Figure 3. The rock rupture experiments seem to support the critical sensitivity: the sensitivity increases significantly prior to the rupture point.

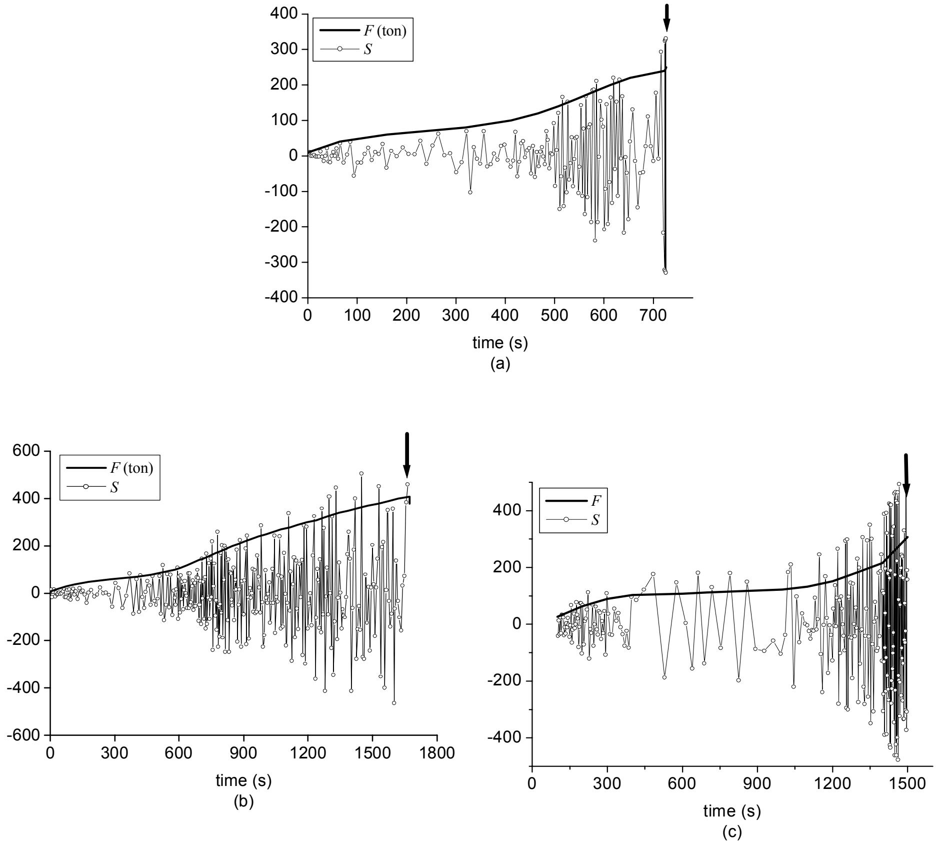

Figure 3: Critical sensitivity in a rock rupture experiment. The curves are corresponding to axial force $F(t)$ and the sensitivity $S(t)$, respectively, and $S(t)$ is calculated from the data of acoustic emission. The total of recorded events is about $4*10^5$. The arrow indicates the catastrophic rupture point. The experimental results seem to support the critical sensitivity: the sensitivity increases significantly prior to the catastrophic rupture point. (a) Gneiss; (b) Sandstone; (c) Marble.

Summary

In summary, the rupture in heterogeneous brittle media presents catastrophe, and the critical sensitivity might be a common precursory feature of catastrophe. This may give a clue to catastrophe prediction.

Acknowledgments

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