

# Quasi-dynamic deterministic and stochastic modeling of earthquake failure at intermediate scales

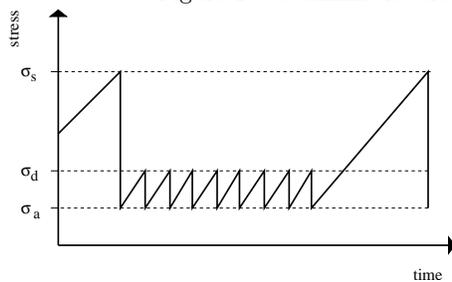
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We present a hierarchical model for simulating earthquake failure at intermediate scales (space: 100 m - 100 km, time: 100 m/ $v_{shear}$ - 1000's of years). The model consists of three hierarchies: the system (top level) as a whole contains a set of faults (middle level); each fault is composed of an array of cells (bottom level). The system is embedded in a three-dimensional elastic half-space as in the framework of Ben-Zion and Rice (1993). At the system level, the interaction between the faults is accounted for. The fault controls the interaction of the cells during an event, while the accumulation and the release of stress takes place on the cell level.

At the present state of model development, only a single rectangular fault is considered. The fault consists of a computational grid of uniform cells, where deformational processes are calculated. Tectonic loading is imposed by a motion with constant velocity  $v_{pl}=35$  mm/year of the regions around the computational grid. The loading rate of each cell varies in relation to the boundary conditions as discussed in Ben-Zion and Rice (1993). The grid of cells is governed by a static/kinetic friction law, i.e., a cell slips initially at each event, if a threshold  $\sigma_s$  is exceeded. The stress drops to an arrest stress  $\sigma_a$  and the threshold decreases instantaneously to the dynamic threshold  $\sigma_d$  and remains there until the earthquake is terminated (see Fig. 1).

Figure 1: deterministic stress loading



The stress transfer during an earthquake is calculated by means of the three-dimensional solution of Chinnery (1963) for static dislocations on rectangular patches in an elastic Poisson solid with rigidity  $\mu = 0.3$ . In particular, we approximate the 3+1 dimensional space time stress transfer by

$$\Delta\sigma(x, t) = \gamma \int G(x, y) \Delta u(y, t - |x - y|/v_s) dy,$$

where  $G$  is the static Green's function,  $\Delta u$  is the slip,  $v_s$  is the shear wave velocity, and  $\gamma$  corresponds to the stress loss term in (Dahmen et al., 1998), which accounts for different rigidities on and off the fault during slip. This approach extends the quasi-static model of Ben-Zion and Rice (1993) to a quasi-dynamic procedure with a finite communication speed ( $v_s$ ) for stress transfer and a related causal rupture process. In one version of the model, the quasi-dynamic rupture process is calculated on a continuous time-scale. This may, however, lead to a numerical explosion during the simulation, for certain parameter values. Therefore, we also study simplifications of the continuous time process, e.g., by discretizing the timescale for the stress transfer during an earthquake.

Furthermore, a stochastic version of the model is considered. To take into account subscale processes, a stochastic component is included on the cell level, such that the decay probability is random, and the decay rate follows

$$\lambda(\sigma) = k \cdot \begin{cases} (\sigma - \sigma_c)^\delta & \text{for } \sigma > \sigma_c \\ 0 & \text{else.} \end{cases}$$

So far, we used fixed values  $k = 1$  and  $\delta = 1.5$ ; a parameter space study is left for future work. Using the parameters from Ben-Zion and Rice (1993), we performed simulations with the different model versions described above. The calculated earthquake catalogs exhibit complex seismicity sequences (Fig. 2). We observe in cases with high stress loss (e.g.,  $\gamma = 0.3$ ), a Gutenberg-Richter scaling relation for the potency  $p = \int \Delta u dA$  ( $A$ =area) and the rupture area (number of slipped cells) over large regions of the potency/area. In particular, the scaling regions in the quasi-dynamic model extend those of the quasi-static model with instantaneous stress transfer (e.g. see Fig. 3). Consequently, the final velocity stress propagation seems to be a relevant feature for the model dynamics (Gabrielov et al., 1994). The stress loss  $\gamma$  controls the size of the largest earthquakes. For larger values of  $\gamma$  the Richter-b-value increases and the size of the largest earthquake decreases (Fig. 4). For low values of  $\gamma$  (e.g.,  $\gamma = 0.1$ ), the statistics resemble the characteristic earthquake distribution. The stochastic model shows a similar scaling like the deterministic model, except for large earthquake sizes, where the Gutenberg-Richter curve is smoother (Fig. 5). Furthermore, a scaling relation  $A \sim p^{0.8}$  between the potency  $p$  and the rupture area  $A$  is found (see Fig. 6). This is close to the empirical relation with an exponent of 2/3 (Kanamori and Anderson, 1975).

In sum, our model reproduces basic features of observed seismicity. The continuing work will deal with refinements of the model (e.g., time dependent healing, realistic heterogeneities) in order to study the spatiotemporal clustering of seismicity. To have a flexible computational environment, we have implemented the model

in a modular C++ class library. We also plan to focus on the dynamics of single ruptures and calculations of synthetic seismograms.

Figure 2: Example for an earthquake sequence

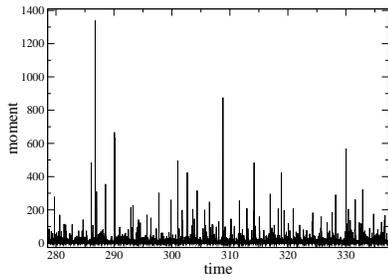


Figure 3: Frequency size distribution with stress loss 0.3

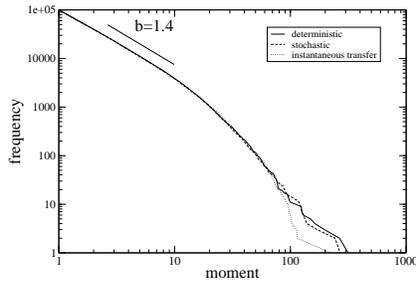


Figure 4: Frequency size distribution for different values of the stress loss

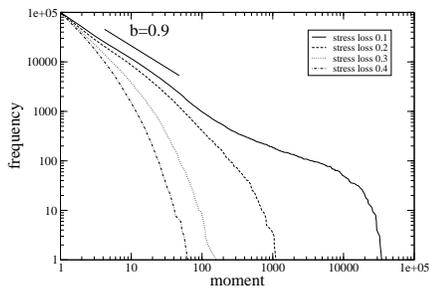


Figure 5: Frequency size distribution with stress loss 0.2

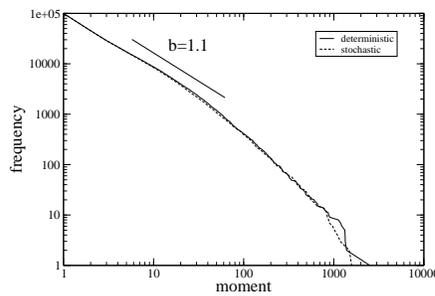
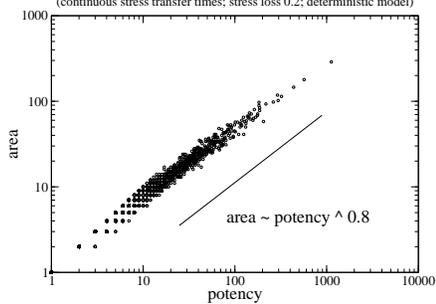


Figure 6: Area moment relation  
(continuous stress transfer times; stress loss 0.2; deterministic model)



## References

- [1] Ben-Zion, Y. and Rice, J. R., 1993, *Earthquake failure sequences along a cellular fault zone in a three-dimensional elastic solid containing asperity and nonasperity regions*, J. Geophys. Res. 98, 14,109-14,131.
- [2] Chinnery, M., 1963, *The stress changes that accompany strike-slip faulting*, Bull. Seim. Soc. Am. 53, 921-932.

- [3] Dahmen, K., Ertas, D. and Ben-Zion, Y., 1998, *Gutenberg-Richter and characteristic earthquake behavior in simple mean-field models of heterogeneous faults*, Phys. Rev. E 58, 1494-1501.
- [4] Gabriellov, A., Newman, W. I. and Knopoff, L., 1994, *Lattice models of failure: Sensitivity to the local dynamics*, Phys. Rev. E 50, 188-197.
- [5] Kanamori, H. and Anderson, D. L., 1975, *Theoretical basis of some empirical relations in seismology*, Bull. Seim. Soc. Am. 65, 1073-1095.