

Simulations of the Burridge-Knopoff Model

Junchao Xia⁽¹⁾, Harvey Gould⁽¹⁾, and W. Klein⁽²⁾

(1) Clark University, Department of Physics, Worcester, MA 01610, e-mail: jcxia@physics.clarku.edu, hgould@clarku.edu. (2) Boston University, Department of Physics and Center for Computational Science, Boston, MA 02215 and Los Alamos National laboratory, Los Alamos, NM 08755. e-mail: klein@buphy.bu.edu.

Abstract

We simulate the Burridge-Knopoff model in one dimension to determine if this dynamical model shows scaling behavior similar to various cellular automata models of earthquake faults. Our main results are that the cluster-size distribution for large clusters (events) is sensitive to the magnitude of the time step, and that the scaling region is poorly defined for nearest-neighbor interactions.

The Burridge-Knopoff Model

We study the behavior of a simple mechanical model of earthquake faults due to Burridge and Knopoff [1] and made popular in the physics community by Carlson and Langer [2]. One reason for the recent interest in dynamical models of this type is that some workers have claimed that the distribution of earthquake-like events exhibits power-law scaling consistent with Gutenberg-Richter.

In the Burridge-Knopoff model, blocks of mass m are coupled by springs of strength κ_c and attached to a fixed surface by springs of strength κ_p . The blocks are in contact with rough substrate that moves at speed v to the left. There is a velocity-dependent frictional force F between the blocks and the rough surface. Newton's equations of motion take the form:

$$m\ddot{x}_j = \kappa_c(x_{j+1} - 2x_j + x_{j-1}) - \kappa_p x_j - F(v + \dot{x}_j), \quad (1)$$

where x_j is the displacement of block j from its equilibrium position. The frictional force F has the functional form

$$F(\dot{x}) = F_0 \phi(\dot{x}/\tilde{v}), \quad (2)$$

with

$$\phi(y) = \frac{1}{1 + |y|} \text{sgn}(y). \quad (3)$$

The speed \tilde{v} characterizes the velocity dependence of F . Note that $\phi(y)$ in Eq. (3) ranges between ± 1 at $y = 0$ and decreases monotonically to zero as y becomes large. This form of the frictional force was adopted in Ref. [1] and is associated with the stick-slip velocity-weakening friction force at the interface between tectonic plates.

It is convenient to introduce the scaled variables

$$\tau = \omega_p t \quad (4)$$

$$x_j = (F_0/\kappa_p)u_j = D_0 u_j, \quad (5)$$

where $\omega_p^2 = \kappa_p/m$. Then we can rewrite Eq. (1) as

$$\ddot{u}_j = \ell^2(u_{j+1} - 2u_j + u_{j-1}) - u_j - \phi(2\alpha\nu + 2\alpha\dot{u}_j), \quad (6)$$

where we have introduced the three dimensionless parameters:

$$\ell^2 = \kappa_c/\kappa_p, \quad \nu = \tilde{v}/(\omega_p D_0), \quad 2\alpha = \omega_p D_0/\tilde{v}. \quad (7)$$

The dots in Eq. (6) denote differentiation with respect to τ .

As discussed in Ref. [2], most of the slipping events that occur in the Burridge-Knopoff model involve connected groups of blocks. To define these events or clusters, we divide the blocks into two classes, those that are “stuck,” that is, have (dimensionless) speeds less than a certain amount v_0 , and blocks that have “slipped” with speeds greater than v_0 . Two slipped blocks that are within the interaction range are in the same cluster. The main physical quantity of interest is the cluster-size distribution, n_s , the normalized number of clusters of s slipped blocks. We take $v_0 = 0.01$ and discuss in the next section why this choice is reasonable.

Results

All of our runs were done with free boundary conditions and for the choice of parameters:

$$\ell = 10, \quad \nu = 0.01, \quad \alpha = 2.5. \quad (8)$$

The same conditions were considered in Ref. [2]. All results shown are averaged over 100 000 τ after running for 10 000 τ to reach a steady state. Data is taken every τ .

Our first job is to choose an appropriate algorithm for solving Eq. (6) numerically and to choose the value of the time step Δt . The usual criterion for the choice of Δt is energy conservation, but because the Burridge-Knopoff model is dissipative, we need another criterion, and we choose Δt to be small enough so that it does not affect the behavior of n_s . Because the fourth-order Runge-Kutta algorithm is a common choice [3], we adopt it first and show our results for n_s in Fig. 1 for different values of Δt . We see that the bump for large s that occurs for $\Delta t = 0.005$ goes away when we make Δt smaller and that the results for $\Delta t = 0.001$ and $\Delta t = 0.0005$ are similar. Although the definition of an event in Ref. [2] differs somewhat from ours, Carlson and Langer found a peak for larger magnitude events. Our results suggest that this peak in n_s is an artifact of their choice of time step or numerical method, neither of which was specified.

Because we need to make long runs to obtain meaningful average, the value $\Delta t = 0.001$ is much smaller than is desirable. We found that the Runge-Kutta-Fehlberg45 algorithm [3, 4] with a fixed time step allows us to choose $\Delta t = 0.005$. Hence, a different choice of algorithm allows us to run almost five times faster. However, more work is necessary to determine the algorithm that is most suitable

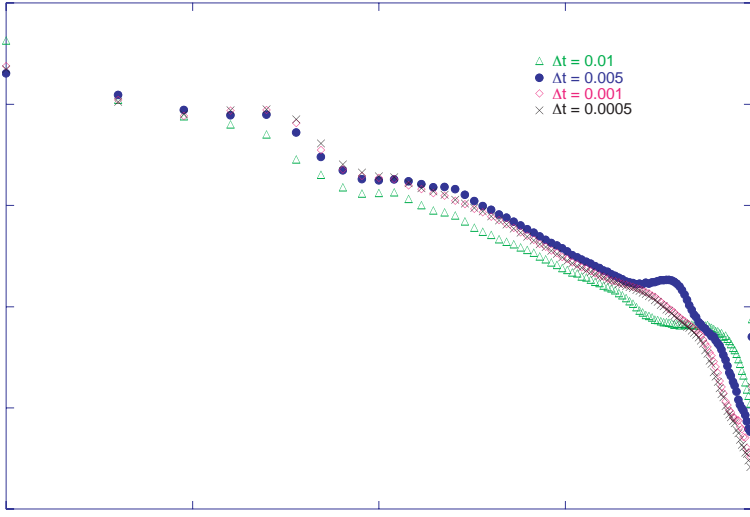


Figure 1: Log-log plot of the cluster size distribution, n_s , versus the number of blocks s for different values of the time step Δt . The equations of motion, Eq. (6), was solved using the RK4 algorithm. Note that $\Delta t \leq 0.001$ is necessary to achieve convergent results. The results are for $N = 100$ blocks.

for a driven, dissipative system such as the Burridge-Knopoff model. We suspect that one reason for the necessity of choosing a small value of Δt is the form of the friction law in Eq. (3) and the jump in $\phi(y)$ for small arguments.

We next determine how many blocks were needed to eliminate finite-size effects. In Fig. 2 we show n_s versus s for different values of N . We see that the results are unchanged for $N \geq 500$ and hence the results for $N = 50$ and 100 as has been considered in the literature should be interpreted with caution.

From the results for n_s shown in Fig. 2, we see that there is a small range of s ($10 \leq s \leq 75$) for which n_s might be said to exhibit power-law scaling. If we fit this range to a straight line, we find that $n_s \sim s^{-a}$ with $a \approx 2.5$. However, this range of s is too limited to conclude that the Burridge-Knopoff model with nearest neighbor interactions exhibits Gutenberg-Richter scaling.

In Fig. 3, we see that the form of n_s is very similar for $v_0 = 0.1$ and 0.01, but differs considerably for larger s for $v_0 = 0.5$ and for smaller s for $v_0 = 0.001$. Because the form of n_s does not vary much in the vicinity of $v_0 = 0.01$, we adopt this value.

Future Work

We have seen that it is necessary to choose a sufficiently small time step and sufficiently large number of blocks to eliminate artifacts of the simulation. Clearly, we need to optimize the numerical algorithm so that larger time steps can be used. Similar studies have been done for conservative systems, but less attention has been given to driven dissipative systems due to the lack of an obvious criterion for choosing one algorithm over another.

Of particular interest is the extension of the Burridge-Knopoff model to longer

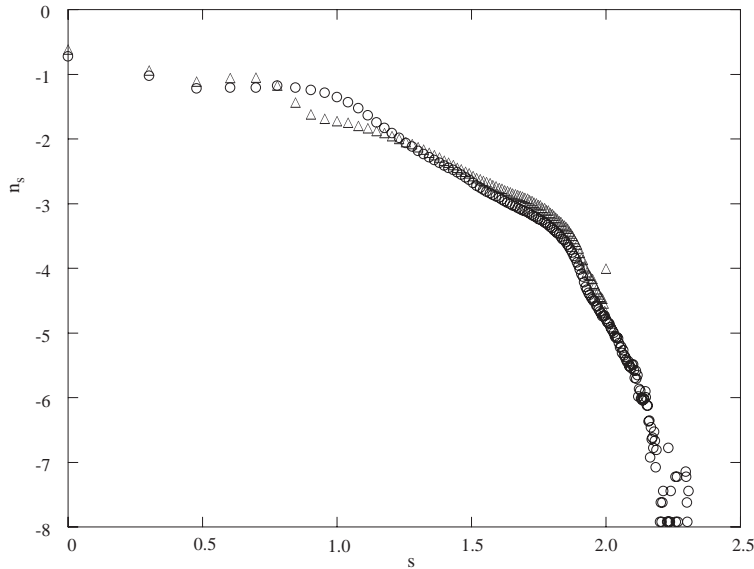


Figure 2: Log-log plot of the cluster size distribution, n_s , versus s for different values of N , the number of blocks, corresponding to $N = 100$ (xx) and $N = 500$ (xx). [xx symbols? xx] To avoid confusion, larger values of N are not shown because they are similar to $N = 500$.

range interactions. That is, instead of one block interacting with two springs (one on each side), we can consider the interaction of one block with $2R$ blocks. In order to go to the mean-field limit corresponding to $R \rightarrow \infty$, we have to scale the strength of the coupling by $1/R$ in one dimension so that the product $R\kappa_c$ is a constant. In this way, we can consider the mean-field limit and make contact with the mean-field limit of various cellular automata earthquake models proposed by Rundle, Jackson, and Brown and Olami, Feder, and Christiansen [5]. Work on these models has shown that n_s does not have a well-defined power-law region for nearest-neighbor interactions [6], but the power-law region becomes better defined, the longer the range of the interaction. It would be interesting to see if a similar longer-range extension of the Burridge-Knopoff model exhibits similar behavior.

Because the Burridge-Knopoff model is an example of a driven dissipative system, its properties are of general interest in statistical physics. In this context it would be interesting to compute various metrics that are a measure of the ergodicity of the system [7] and to compute the form of the energy distribution. Of particular interest, its how the metrics and various distributions behave as the range of the block-block interaction is increased.

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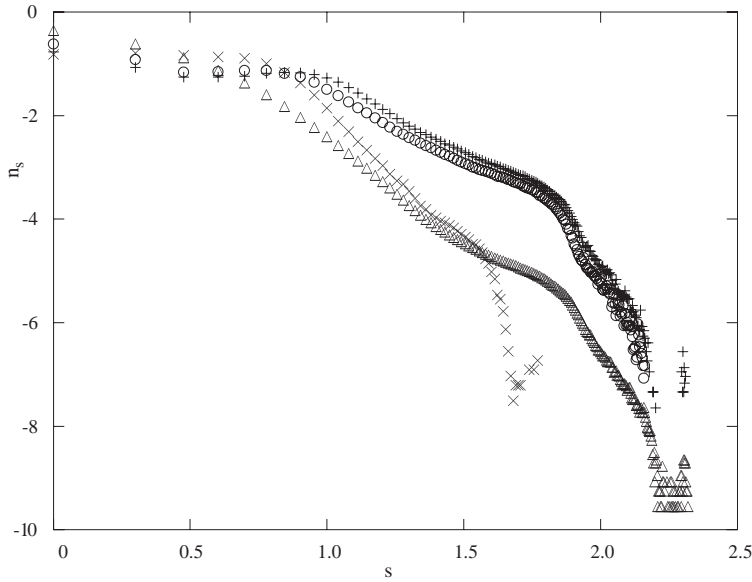


Figure 3: Log-log plot of n_s versus s for different values of v_0 , the velocity cutoff, corresponding to $v_0 = 0.001$ (\triangle), 0.01 ($+$), 0.1 (\circ), and 0.5 (\times). Note that n_s is similar for $v_0 = 0.01$ and 0.1 . For these runs $N = 1499$.

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