Aftershocks and damage mechanics

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Abstract

Aftershock sequences are universal consequences of earthquakes. The time delays associated with aftershocks can be explained in terms of damage mechanics. If a strain is applied instantaneously so that the stress is larger than the critical stress, microcracking relieves this stress until it is reduced to the critical value. A main shock increases the strain in some adjacent regions. The aftershocks relieve these increases in stress just as microckracks relieve stress in the damage model. The solution of the damage mechanics model for the instantaneous application of strain reproduces Omori's law and other observed aspects of aftershocks sequences.

Introduction.

Any deviation of a solid from reversible elastic behavior is often referred to as *damage*. Damage includes the thermally activated creep processes (diffusion and dislocation creep) responsible for mantle convection. Damage also includes the plastic deformation of ductile materials and the rupture of brittle materials. In this paper we will concentrate our attention on the irreversible deformation of brittle solids with the object of better understanding the deformation of the earth's crust.

Many experiments on the fracture of brittle solids have been carried out. In terms of rock failure, the early experiments by Mogi (1962) were pioneering. Acoustic emissions (AE) associated with microcracks were monitored, Gutenberg-Richter frequency-magnitude statistics were observed for AE. When a load was applied very rapidly, the time-to-failure was found to depend on the load. Many other studies of this type have been carried out. Otani et al. (1991) obtained the statistical distribution of the life times with constant stress loading for carbon fibre-epoxy microcomposites. Johansen and Sornette (2000) studied the rupture of spherical tanks of kevlar wrapped around thin metallic linens and found a power-law increase of AE prior to rupture. Guarino et al. (1998, 1999) studied the failure of chipboard panels. They obtained power-law increases in AE prior to rupture and a systematic dependence of failure times on stress level.

Statistical physicists have related brittle rupture to liquid-vapour phase changes in a variety of ways. Buchel and Sethna (1997) have associated brittle rupture with a first-order phase transition. Similar arguments have been given by Zapperi et al. (1997) and Kun and Herrmann (1999). On the other hand Sornette and Andersen (1998) and Gluzman and Sornette (2001) argue that brittle rupture is analogous to a critical point phenomena, not to a first-order phase change. They associated observed power-law scaling in brittle failure experiments with a critical point (second-order phase change). An approach that unites the above views is to consider brittle rupture in analogy to spinodal nucleation (Selinger et al. 1991; Rundle et al. 1996, 1999; Zapperi et al. 1999). We will consider this analogy in more detail in the next section.

A characteristic of the rupture of brittle materials, as described above, is the time delay between the application of a stress and the rupture. In the earth's brittle crust, the aftershock sequences following all earthquakes are examples of this type of delay. When a rupture occurs on a fault there is a redistribution of stress around the fault. Some regions have a reduction in stress (stress shadows), and other regions have an increase in stress (stress halos). Since the total stored elastic energy must decrease, the integrated reduction of stress is greater than the integrated increase. Nevertheless, the regions of increased stress are significant and it is these regions where aftershocks occur (both on the fault that initially ruptured and on adjacent faults) (Dieterich 1994).

A universal scaling law is applicable to the temporal decay of aftershock activity following an earthquake. This is known as Omori's law and has the form (Scholz 1990)

$$\frac{dn_{\mathrm{as}}}{dt} = \frac{C_1}{[C_2 + (t - t_{\mathrm{ms}})]^p}\,, \tag{1}$$

where n_{as} is the number of aftershocks with magnitudes greater than a specified value, $t - t_{\mathrm{ms}}$ is time measured forward from the occurrence of the mainshock at t_{ms}, C_1 and C_2 are constants, and the power p has a value near unity. We will show that damage mechanics predicts a power-law decay of aftershocks following the near instantaneous application of stress during a main shock.

Damage mechanics

In order to provide a basis for discussing material failure as a phase change process, we first discuss the phase diagram for the coexistence of the liquid and vapour phases of a pure substance. A schematic pressure-volume projection of a phase diagram is illustrated in Figure 1a (Debenedetti 1996). We consider the boiling of a liquid initially at point A in the figure. The pressure is decreased isothermally until the phase change boundary is reached at point B. In thermodynamic equilibrium the liquid will boil at constant pressure and temperature until it is entirely a vapour at point G. Further reduction of pressure will result in the isothermal expansion of the vapour along the path GF. However, it is possible to create a metastable, superheated liquid at point B. If bubbles of vapour do not form, either by homogeneous or heterogeneous nucleation, the liquid can be superheated along the path BD. The point D is the intersection of the liquid P–V curve with the spinodal curve S. It is not possible to superheat the liquid beyond this point. If the liquid is superheated to the vicinity of point D, explosive nucleation and boiling will take place. If the pressure and temperature are maintained constant during this highly nonequilibrium explosion, the substance will follow the path DE to the vapour equilibrium curve GF. If the explosion occurs at constant volume and temperature, the pressure will increase as the substance follows the path DH to the equilibrium boiling line BG. Any horizontal path between the superheated liquid BD and vapour GE is possible.

An example is the path *IJ*. The entire shaded region is metastable. A point on horizontal lines, for example *IJ*, is determined by the "wetness" of the liquid-vapour mixture, the mass fraction that is liquid. At point I the mass fraction of liquid is 1, at point J the mass fraction of liquid is 0. Which path is followed in the metastable region is determined by the physics of the bubble nucleation processes (Debenedetti 1996).

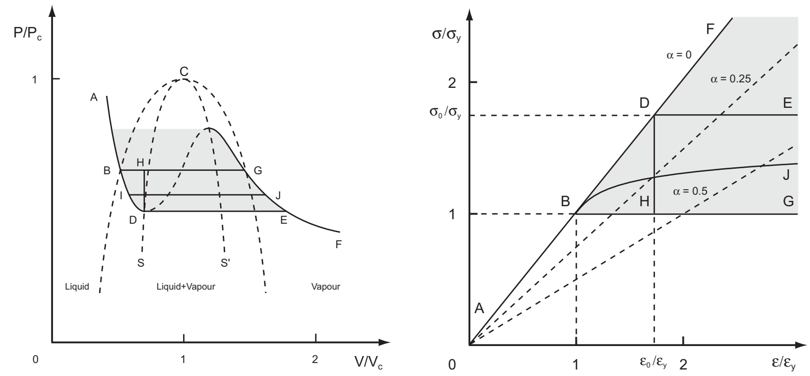

Figure 1: (a) Schematic pressure-volume ($P - V$) projection of the phase diagram of a pure substance. The shaded region is metastable. (b) Idealized stress-strain diagram for a brittle solid.

We next apply the concept of phase change to the brittle fracture of a solid. For simplicity we will discuss the failure of a sample of area a under compression by a force F. The state of the sample is specified by the stress $\sigma = F/a$ and its strain $\epsilon = (L - L_0)/L_0$ (L length, L_0 initial length). The dependence of the stress on strain is illustrated schematically in Figure 1b.

We hypothesize that a prestine brittle solid will obey linear elasticity (Hooke's law) for stresses in the range $0 \le \sigma \le \sigma_y$, where σ_y is a yield stress. The corresponding yield strain ϵ_y is $\epsilon_y = \sigma_y/E_0$. If stress is applied infinitely slowly (to maintain a thermodynamic equilibrium), we further hypothesize that the solid will fail at the yield stress σ_y. The failure path ABG in Figure 1b corresponds to the equilibrium failure path ABG in Figure 1a. At stresses greater than the yield stress, $\sigma > \sigma_y$, damage occurs in the form of microcracks. This damage results in accelerated strain and a deviation from linear elasticity. A typical failure path ABJ is illustrated in Figure 1b. In order to quantify the deviation from linear elasticity the damage variable α has been introduced in the strain-stress relation

$$\sigma = E_0\,(1 - \alpha)\,\epsilon\,. \tag{2}$$

When $\alpha = 0$, (2) reduces to $\sigma = E_0\,\epsilon$ and linear elasticity is applicable; as $\alpha \to 1$, $\epsilon \to \infty$ failure occurs.

Based upon thermodynamic considerations (Kachanov 1986; Krajcinovic 1996; Lyakhovsky et al. 1997), the time evolution of the damage variable is related to the time dependent stress $\sigma(t)$ and strain $\epsilon(t)$ by

$$\frac{d\alpha(t)}{dt} = A\,(\sigma(t))\,\left[\frac{\epsilon(t)}{\epsilon_y}\right]^2\,. \tag{3}$$

It should be noted that there are alternative formulations of both (2) and (3) and that $A(\sigma)$ can take many forms (Krajcinovic 1996). In our analysis we will assume (2) and (3) are applicable and will further require that

$$A\left(\sigma(t)\right) = 0 \quad \text{if} \quad 0 \leq \sigma \leq \sigma_{\mathrm{y}} \tag{4}$$

$$A\left(\sigma(t)\right) = \frac{1}{t_d}\left[\frac{\sigma(t)}{\sigma_y} - 1\right]^{\rho} \quad \text{if} \quad \sigma > \sigma_{\mathrm{y}}\,, \tag{5}$$

where t_d is a characteristic time scale for damage and ρ is a power to be determined from experiments.

The monotonic increase in the damage variable α given by (3)–(5) represents the weakening of the brittle solid due to the nucleation and coalescence of microcracks. This nucleation and coalescence of microcracks is analogous to the nucleation and coalescence of bubbles in a superheated liquid as discussed above. A brittle solid in region *GBF* in the stress-strain diagram given in Figure 1b is metastable in the same sense that the nonequilibrium boiling in region *BDEG* in Figure 1a is metastable.

Solutions to one-dimensional damage problems require the simultaneous solution of (2)–(5). In general it is required that either the stress or the strain on the sample be specified.

Constant applied strain

As an analogy to aftershocks we consider a rod to which a constant axial compressive strain has been applied. A strain $\epsilon_0 > \epsilon_y$ is applied instantaneously at $t = 0$ and is held constant. The applicable equation for the rate of increase of damage is obtained from (2), (3) and (5) with the result

$$\frac{d\alpha}{dt} = \frac{1}{t_d}\left(\frac{\epsilon_0}{\epsilon_y}\right)^2 \left\{\frac{\epsilon_0}{\epsilon_y}\left[1 - \alpha(t)\right] - 1\right\}^{\rho}. \tag{6}$$

Integrating with the initial condition $\alpha(0) = 0$, we find

$$\alpha(t) = 1 - \frac{\epsilon_y}{\epsilon_0}\left\{1 + \left[\left(\frac{\epsilon_0}{\epsilon_y} - 1\right)^{-(\rho-1)} + (\rho - 1)\left(\frac{\epsilon_0}{\epsilon_y}\right)^3\left(\frac{t}{t_d}\right)\right]^{-\frac{1}{\rho-1}}\right\}. \tag{7}$$

The damage increases monotonically with time and as $t \to \infty$ the maximum damage is $\alpha(\infty) = 1 - \frac{\epsilon_y}{\epsilon_0}$. Using (2) and (7) one gets the stress relaxation in the material as a function of time t

$$\frac{\sigma(t)}{\sigma_y} = 1 + \left[\left(\frac{\epsilon_0}{\epsilon_y} - 1\right)^{-(\rho-1)} + (\rho - 1)\left(\frac{\epsilon_0}{\epsilon_y}\right)^3\left(\frac{t}{t_d}\right)\right]^{-\frac{1}{\rho-1}}. \tag{8}$$

At $t = 0$ we recover $\sigma(0) = E_0\,\epsilon_0$, which is the stress corresponding to the strain ϵ_0 from the linear elastic relation as expected. In the limit $t \to \infty$ we have $\sigma(\infty) = \sigma_y$. The stress relaxes to the yield stress σ_y below which no further damage can occur, again as expected. The nondimensional stress $\sigma(t)/\sigma_y$ from (8) is given as a function of nondimensional time t/t_d in Figure 2a taking $\rho = 2$ and several values of the applied nondimensional strain ϵ_0/ϵ_y.

We believe that this stress relaxation process is applicable to the understanding of the aftershock sequence that follows an earthquake. During an earthquake some regions in the vicinity of the earthquake experience a rapid increase of stress (strain). This is in direct analogy to the rapid increase in strain considered in this Section. However, the stress σ is greater than the yield stress σ_y and microcracks (aftershocks) relax the stress to σ_y just as described above. The time delay of the aftershocks relative to the main shock is in direct analogy to the time delay of the damage. This delay is because it takes time to nucleate microcracks (aftershocks).

In order to quantify the rate of aftershock occurrence we determine the rate of energy release in the relaxation process considered above. The energy density (per unit mass) e_0 in the rod after the instantaneous strain has been applied is $e_0 = E_0\,\epsilon_0^2/2$.

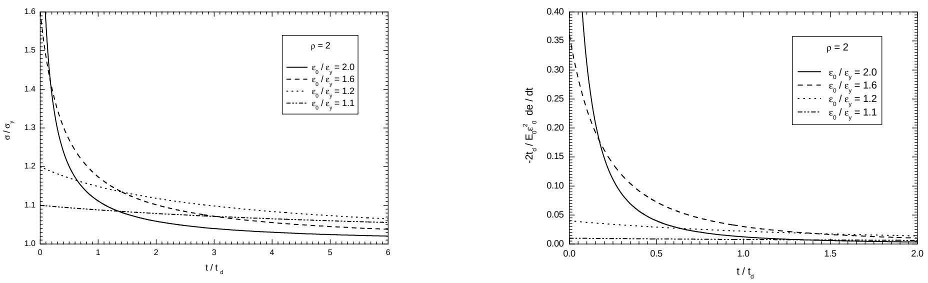

Figure 2: (a) Stress relaxation after the instantaneous application of a constant strain ϵ_0 that exceeds the yield strain ϵ_y. (b) The relaxation of the energy rate.

We hypothesize that if during the relaxation process, the stress is instantaneously removed, the sample will be returned to a state of zero stress and strain following a linear stress-strain path with slope $E_0\,(1-\alpha)$. With this assumption the energy density in the rod during the stress relaxation is given by

$$e = \frac{E_0\,\epsilon_0^2}{2}\,(1-\alpha)\,. \tag{9}$$

The rate of energy release is obtained by substituting (7) into (9) and taking the time derivative with the result

$$\frac{de}{dt} = -\frac{E_0\,\epsilon_0^2}{2\,t_d}\left[\left(\frac{\epsilon_0}{\epsilon_y}-1\right)^{-(\rho-1)} + (\rho-1)\left(\frac{\epsilon_0}{\epsilon_y}\right)^3\left(\frac{t}{t_d}\right)\right]^{-\frac{\rho}{\rho-1}}\,. \tag{10}$$

Taking $\rho/(\rho-1) = p$ this has the form of Omori's law given in (1). If ρ is large then $p \to 1$ in accordance with observations. However, it should be noted that we have derived the rate of energy release whereas Omori's law is the rate of aftershock occurrence. The energy release is dominated by the largest aftershocks. Thus it may decay more rapidly that frequency rate. In any case the power law decay is obtained from our damage analysis.

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